Exam 1 (Total 100 points)

1. True/False (4 points each)
   a) T $p \rightarrow q \equiv \neg q \rightarrow \neg p$.
   b) T $\forall x\, P(x) \equiv \neg\exists x \neg P(x)$.
   c) F The following is a valid argument:

      $p \rightarrow q$
      q
      ----
      p.

   d) F $\emptyset \in \emptyset$
   e) F The cardinality of the power set of {a, b, c} is 9.

2. Briefly define the following terms (4 points each)
   a) An injective (one to one) function.
      (different elements in domain have distinct images)
   b) Cartesian product of two sets.
      (a set of ordered pairs, …)
   c) Proposition in propositional logic.
      (a statement with truth value)
   d) $f(x)$ is $O(g(x))$.
      ($\exists C,k$: $f(x) < Cg(x)$ for all $x > k$).)
   e) Range of a function.
      (set of all images of a function.)

3. Briefly answer the following short questions (10 points each)
   a) Determine if the following argument is valid and explain why.
      Every CS major takes CMSC203. Joel is taking CMSC203, therefore he is a CS major.
      Invalid: $p \rightarrow q$ and q do not support the conclusion of p
   b) What is the time complexity (in Big-O notation) of the following code? Your answer should be a function of n. Also, what is the value of "sum" after the execution of the code with n = 10?

```
sum := 0;
for i := 1 to n
   for j := i to n
      sum := sum + 1.
```

      $O(n^2)$, sum = 55.

4. (20 points) Construct an argument using rules of inference to show that the hypotheses
   - All humans are mortal,
   - All philosophers are humans, and
   - Socrates is a Greek philosopher

   imply the conclusion that
   - Socrates is mortal.

Predicates: H(x), M(x), P(x), and G(x), for x is a human, x is mortal, x is a philosopher, and x is Greek, respectively.

Step 1: $\forall x(H(x) \rightarrow M(x))$	hypothesis
Step 2: $\forall x(P(x) \rightarrow H(x))$	hypothesis
Step 3: G(Socrates) $\wedge$ P(Socrates)	hypothesis
Step 4: P(Socrates)	simplification on Step 3
Step 5: P(Socrates) $\rightarrow$ H(Socrates)	universal instantiation on Step 2
Step 6: H(Socrates)	modes ponens on Steps 4 & 5
Step 7: H(Socrates) $\rightarrow$ M(Socrates)	universal instantiation on Step 1
Step 8: M(Socrates)	Modes ponens on Steps 6 & 7

5. (10 points) Use direct proof to show that there exists a rational number in between any two distinct rational numbers.

   Any two distinct rational numbers x and y can be written as $x = a/b$, $y = c/d$, where a, b, c, and d are integers (and b and d are not zero). Then $z = (x + y)/2 = (ad + bc)/2bd$ is a rational number in between x and y.

6. (10 points) Is the following statement true?

   If $A \cup C = B \cup C$ then $A = B$, where A, B, and C are three sets.

   If yes, give a proof, otherwise find a counterexample.

   The statement is false. Counterexample:

   Let A = {1,2}, B = {3, 4}, C = {1,2,3,4}. Then we have

   $A \cup C = B \cup C$ = {1,2,3,4} but A != B.