

Question 1: Determine the types of the following functions

Determine the types of the following functions:

- a) `fun triple x = 3 * x;`
 - b) `fun compose f g x = f (g x);`
 - c) `fun filterEven [] = [] | filterEven (x::xs) = if x mod 2 = 0 then x::(filterEven xs) else filterEven xs;`
 - d) `fun pairwise f (a, b) = (f a, f b);`
 - e) `fun map2 f [] [] = [] | map2 f (x::xs) (y::ys) = (f x y)::(map2 f xs ys);`
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Question 2: Implement a Stack Signature

signature STACK = sig

(* A type for a stack with elements of type 'a *)

`type 'a stack`

(* Creates an empty stack *)

`val empty: unit -> 'a stack`

(* Pushes an element onto the stack *)

`val push: 'a * 'a stack -> 'a stack`

(* Removes the top element from the stack. Raises Empty if the stack is empty. *)

`val pop: 'a stack -> 'a stack`

(* Returns the top element of the stack without removing it. Raises Empty if the stack is empty. *)

`val top: 'a stack -> 'a`

(* Checks if the stack is empty *)

`val isEmpty: 'a stack -> bool`

(* Applies a function to each element of the stack, from top to bottom, and returns a new stack with the results *)

```
val map: ('a -> 'b) -> 'a stack -> 'b stack
```

```
end;
```

Implementation Task:

1. Define a structure that matches the `STACK` signature.
 2. Implement the stack functionality using a list to store elements.
 3. Ensure your implementation correctly handles empty stack cases.
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Test Cases:

After implementing the `STACK` structure, create test cases that cover each of the following scenarios:

- Creating an empty stack and checking if it is indeed empty.
- Pushing one element, then multiple elements, and verifying the state of the stack after each operation.
- Popping elements from the stack until it is empty, checking the correctness of each removed element, and ensuring the `Empty` exception is raised when attempting to pop from an empty stack.
- Checking the top element of the stack without removing it and ensuring the `Empty` exception is raised when checking an empty stack.
- Testing the `isEmpty` function on both empty and non-empty stacks.
- Applying a `map` function to a stack, such as squaring numbers in a stack of integers, and verifying the output stack reflects the applied function without altering the original stack's state.

Submit your code including all the test cases.