CMSC 313 COMPUTER ORGANIZATION & ASSEMBLY LANGUAGE PROGRAMMING

LECTURE 19, SPRING 2013

TOPICS TODAY

- Introduction to Digital Logic
- Semiconductors, Transistors & Gates

INTRODUCTION TO DIGITAL LOGIC

Chapter 3 Objectives

- Understand the relationship between Boolean logic and digital computer circuits.
- Learn how to design simple logic circuits.
- Understand how digital circuits work together to form complex computer systems.

Some Definitions

- Combinational logic: a digital logic circuit in which logical decisions are made based only on combinations of the inputs. *e.g.* an adder.
- Sequential logic: a circuit in which decisions are made based on combinations of the current inputs as well as the past history of inputs. *e.g.* a memory unit.
- Finite state machine: a circuit which has an internal state, and whose outputs are functions of both current inputs and its internal state. e.g. a vending machine controller.

Principles of Computer Architecture by M. Murdocca and V. Heuring

The Combinational Logic Unit

- Translates a set of inputs into a set of outputs according to one or more mapping functions.
- Inputs and outputs for a CLU normally have two distinct (binary) values: high and low, 1 and 0, 0 and 1, or 5 V and 0 V for example.
- The outputs of a CLU are strictly functions of the inputs, and the outputs are updated immediately after the inputs change. A set of inputs i₀ i_n are presented to the CLU, which produces a set of outputs according to mapping functions f₀ f_m.



Principles of Computer Architecture by M. Murdocca and V. Heuring

A-4

Ripple Carry Adder

 Two binary numbers A and B are added from right to left, creating a sum and a carry at the outputs of each full adder for each bit position.



Principles of Computer Architecture by M. Murdocca and V. Heuring

3-6

Classical Model of a Finite State Machine

An FSM is composed of a composed of a combinational logic unit and delay elements (called *flip-flops*) in a feedback path, which maintains state information.



Vending Machine State Transition Diagram



A-71



- Boolean algebra is a mathematical system for the manipulation of variables that can have one of two values.
 - In formal logic, these values are "true" and "false."
 - In digital systems, these values are "on" and "off,"
 1 and 0, or "high" and "low."
- Boolean expressions are created by performing operations on Boolean variables.
 - Common Boolean operators include AND, OR, and NOT.

- A Boolean operator can be completely described using a truth table.
- The truth table for the Boolean operators AND and OR are shown at the right.
- The AND operator is also known as a Boolean product. The OR operator is the Boolean sum.

X AND Y





- The truth table for the Boolean NOT operator is shown at the right.
- The NOT operation is most often designated by an overbar. It is sometimes indicated by a prime mark (') or an "elbow" ([¬]).



- A Boolean function has:
 - At least one Boolean variable,
 - At least one Boolean operator, and
 - At least one input from the set {0,1}.
- It produces an output that is also a member of the set {0,1}.

Now you know why the binary numbering system is so handy in digital systems.

• The truth table for the Boolean function:

 $F(x, y, z) = x\overline{z}+y$

is shown at the right.

 To make evaluation of the Boolean function easier, the truth table contains extra (shaded) columns to hold evaluations of subparts of the function. $F(x, y, z) = x\overline{z} + y$

x	У	z	ī	хī	xīz+y
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	0	1

- As with common arithmetic, Boolean operations have rules of precedence.
- The NOT operator has highest priority, followed by AND and then OR.
- This is how we chose the (shaded) function subparts in our table.

 $F(x, y, z) = x\overline{z}+y$

x	У	z	z	хī	xīz+y
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	0	1

- Digital computers contain circuits that implement Boolean functions.
- The simpler that we can make a Boolean function, the smaller the circuit that will result.
 - Simpler circuits are cheaper to build, consume less power, and run faster than complex circuits.
- With this in mind, we always want to reduce our Boolean functions to their simplest form.
- There are a number of Boolean identities that help us to do this.

 Most Boolean identities have an AND (product) form as well as an OR (sum) form. We give our identities using both forms. Our first group is rather intuitive:

Identity	AND	OR
Name	Form	Form
Identity Law Null Law Idempotent Law Inverse Law	$1x = x$ $0x = 0$ $xx = x$ $x\overline{x} = 0$	0 + x = x 1 + x = 1 x + x = x $x + \overline{x} = 1$

• Our second group of Boolean identities should be familiar to you from your study of algebra:

Identity	AND	OR
Name	Form	Form
Commutative Law	xy = yx	x+y = y+x
Associative Law	(xy) z = x (yz)	(x+y)+z = x + (y+z)
Distributive Law	x+yz = (x+y) (x+z)	x (y+z) = xy+xz

- Our last group of Boolean identities are perhaps the most useful.
- If you have studied set theory or formal logic, these laws are also familiar to you.

Identity Name	AND Form	OR Form
Absorption Law DeMorgan's Law	x (x+y) = x (xy) = x + y	
Double Complement Law	$\overline{(\overline{\mathbf{x}})} = \mathbf{x}$	

• We can use Boolean identities to simplify:

```
F(X, Y, Z) = (X+Y)(X+\overline{Y})(X\overline{Z})
```

as follows:

$(X + Y) (X + \overline{Y}) (\overline{X\overline{Z}})$	
$(X + Y) (X + \overline{Y}) (\overline{X} + Z)$	DeMorgan's Law
	Double complement Law
$(XX + X\overline{Y} + YX + Y\overline{Y}) (\overline{X} + Z)$	Distributive Law
$((X + Y\overline{Y}) + X(Y + \overline{Y}))(\overline{X} + Z)$	Commutative and Distributive Laws
$((X + 0) + X(1))(\overline{X} + Z)$	Inverse Law
$X(\overline{X} + Z)$	Idempotent and Identity Laws
$x\overline{x} + xz$	Distributive Law
0 + XZ	Inverse Law
XZ	Identity Law

- Sometimes it is more economical to build a circuit using the complement of a function (and complementing its result) than it is to implement the function directly.
- DeMorgan's law provides an easy way of finding the complement of a Boolean function.
- Recall DeMorgan's law states:

$$(\overline{xy}) = \overline{x} + \overline{y}$$
 and $(\overline{x+y}) = \overline{x}\overline{y}$

- DeMorgan's law can be extended to any number of variables.
- Replace each variable by its complement and change all ANDs to ORs and all ORs to ANDs.
- Thus, we find the the complement of:

$$F(X, Y, Z) = (XY) + (\overline{XY}) + (\overline{XZ})$$

is:

$$\overline{F}(X, Y, Z) = \overline{(XY) + (\overline{X}Z) + (Y\overline{Z})}$$
$$= \overline{(\overline{XY})}(\overline{\overline{X}Z})(\overline{Y\overline{Z}})$$
$$= (\overline{X} + \overline{Y})(X + \overline{Z})(\overline{Y} + Z)$$

- Through our exercises in simplifying Boolean expressions, we see that there are numerous ways of stating the same Boolean expression.
 - These "synonymous" forms are *logically equivalent*.
 - Logically equivalent expressions have identical truth tables.
- In order to eliminate as much confusion as possible, designers express Boolean functions in standardized or canonical form.

- There are two canonical forms for Boolean expressions: sum-of-products and product-of-sums.
 - Recall the Boolean product is the AND operation and the Boolean sum is the OR operation.
- In the sum-of-products form, ANDed variables are ORed together.

- For example: F(x, y, z) = xy + xz + yz

 In the product-of-sums form, ORed variables are ANDed together:

- For example: F(x, y, z) = (x+y)(x+z)(y+z)

- It is easy to convert a function to sum-of-products form using its truth table.
- We are interested in the values of the variables that make the function true (=1).
- Using the truth table, we list the values of the variables that result in a true function value.
- Each group of variables is then ORed together.

$$F(x, y, z) = x\overline{z}+y$$

x	У	z	xz+y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

• The sum-of-products form for our function is:

$$F(x,y,z) = (\overline{x}y\overline{z}) + (\overline{x}yz) + (x\overline{y}\overline{z}) + (x\overline{y}\overline{z}) + (x\overline{y}\overline{z}) + (x\overline{y}z)$$

We note that this function is not in simplest terms. Our aim is only to rewrite our function in canonical sum-of-products form.

$$F(x,y,z) = x\overline{z}+y$$

xyz
$$x\overline{z}+y$$
000001010011101101110111111111

- We have looked at Boolean functions in abstract terms.
- In this section, we see that Boolean functions are implemented in digital computer circuits called gates.
- A gate is an electronic device that produces a result based on two or more input values.
 - In reality, gates consist of one to six transistors, but digital designers think of them as a single unit.
 - Integrated circuits contain collections of gates suited to a particular purpose.

The three simplest gates are the AND, OR, and NOT gates.



• They correspond directly to their respective Boolean operations, as you can see by their truth tables.

- Another very useful gate is the exclusive OR (XOR) gate.
- The output of the XOR operation is true only when the values of the inputs differ.



Note the special symbol ⊕ for the XOR operation.

 NAND and NOR are two very important gates.
 Their symbols and truth tables are shown at the right.



NAND and NOR are known as universal gates because they are inexpensive to manufacture and any Boolean function can be constructed using only NAND or only NOR gates.



- Gates can have multiple inputs and more than one output.
 - A second output can be provided for the complement of the operation.
 - We'll see more of this later.



- The main thing to remember is that combinations of gates implement Boolean functions.
- The circuit below implements the Boolean function: F(X,Y,Z) = X+YZ



We simplify our Boolean expressions so that we can create simpler circuits.

Sum-of-Products Form: The Majority Function

• The SOP form for the 3-input majority function is:

 $\mathbf{M} = \overline{\mathbf{A}\mathbf{B}\mathbf{C}} + \overline{\mathbf{A}\mathbf{B}\mathbf{C}} + \overline{\mathbf{A}\mathbf{B}\mathbf{C}} + \overline{\mathbf{A}\mathbf{B}\mathbf{C}} = \mathbf{m}\mathbf{3} + \mathbf{m}\mathbf{5} + \mathbf{m}\mathbf{6} + \mathbf{m}\mathbf{7} = \Sigma (\mathbf{3}, \mathbf{5}, \mathbf{6}, \mathbf{7}).$

- Each of the 2ⁿ terms are called *minterms*, ranging from 0 to 2ⁿ 1.
- Note relationship between minterm number and boolean value.





A balance tips to the left or right depending on whether there are more 0's or 1's.

Principles of Computer Architecture by M. Murdocca and V. Heuring

© 1999 M. Murdocca and V. Heuring

AND-OR Implementation of Majority

Gate count is
 8, gate input
 count is 19.



Principles of Computer Architecture by M. Murdocca and V. Heuring

© 1999 M. Murdocca and V. Heuring
Sum of Products (a.k.a. disjunctive normal form)

- OR (i.e., sum) together rows with output 1
- AND (i.e., product) of variables represents each row
 e.g., in row 3 when x₁ = 0 AND x₂ = 1 AND x₃ = 1
 or when x₁ · x₂ · x₃ = 1
- MAJ3 $(x_1, x_2, x_3) = \overline{x_1}x_2x_3 + x_1\overline{x_2}x_3 + x_1x_2\overline{x_3} + x_1x_2x_3 = \sum m(3, 5, 6, 7)$

	x_1	x_2	x_3	MAJ3
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

Product of Sums (a.k.a. conjunctive normal form)

- AND (i.e., product) of rows with output 0
- OR (i.e., sum) of variables represents negation of each row e.g., NOT in row 2 when x₁ = 1 OR x₂ = 0 OR x₃ = 1 or when x₁ + x₂ + x₃ = 1
- MAJ3 $(x_1, x_2, x_3) = (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x_3})(x_1 + \overline{x_2} + x_3)(\overline{x_1} + x_2 + x_3)$ = $\prod M(0, 1, 2, 4)$

	x_1	x_2	x_3	MAJ3
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

OR-AND Implementation of Majority



Principles of Computer Architecture by M. Murdocca and V. Heuring

A-21

© 1999 M. Murdocca and V. Heuring

- Every Boolean function can be written as a truth table
- Every truth table can be written as a Boolean formula (SOP or POS)
- Every Boolean formula can be converted into a combinational circuit
- Every combinational circuit is a Boolean function
- Later you might learn other equivalencies: finite automata ≡ regular expressions computable functions ≡ programs

- Every Boolean function can be written as a Boolean formula using AND, OR and NOT operators.
- Every Boolean function can be implemented as a combinational circuit using AND, OR and NOT gates.
- Since AND, OR and NOT gates can be constructed from NAND gates, NAND gates are universal.

All-NAND Implementation of OR

NAND alone implements all other Boolean logic gates.



© 1999 M. Murdocca and V. Heuring

A-17

DeMorgan's Theorem

A B	\overline{AB} =	$\overline{A} + \overline{B}$	$\overline{A + B}$	$=\overline{A}\overline{B}$
0 0	1	1	1	1
0 1	1	1	0	0
1 0	1	1	0	0
1 1	0	0	0	0

DeMorgan's theorem: $A + B = \overline{\overline{A + B}} = \overline{\overline{A B}}$

$$A \longrightarrow F = A + B \implies A \longrightarrow B \longrightarrow F = \overline{A \ \overline{B}}$$

Principles of Computer Architecture by M. Murdocca and V. Heuring

SEMICONDUCTORS, TRANSISTORS & GATES

How do we make gates???

UMBC, CMSC313, Richard Chang <chang@umbc.edu>

A Truth Table

- Developed in 1854 by George Boole.
- Further developed by Claude Shannon (Bell Labs).
- Outputs are computed for all possible input combinations (how many input combinations are there?)
- Consider a room with two light switches. How must they work?



In	puts	Output
A	В	Ζ
0	0	0
0	1	1
1	0	1
1	1	0

Principles of Computer Architecture by M. Murdocca and V. Heuring

© 1999 M. Murdocca and V. Heuring

A-5

Electrically Operated Switch

• Example: a relay





source: http://www.howstuffworks.com/relay.htm

UMBC, CMSC313, Richard Chang <chang@umbc.edu>

Semiconductors

- Electrical properties of silicon
- Doping: adding impurities to silicon
- Diodes and the P-N junction
- Field-effect transistors

Los Alamos National Laboratory's Chemistry Division Presents

Periodic Table of the Elements

Group

Period	l 1																	18
	IA																	vIIIA
	1A																	8A
	1	2											13	14	15	16	17	2
1	<u>H</u>	IIA											IIIA	IVA	VA	VIA	VIIA	<u>He</u>
	1.008	2A											3A	4A	5A	6A	7A	4.003
•	3	4											5	6	7	8	9	10
2	<u>L1</u>	<u>Be</u>											<u>B</u>	<u>C</u>	<u>N</u>	\underline{O}	<u>F</u>	<u>Ne</u>
	6.941	9.012											10.81	12.01	14.01	16.00	19.00	20.18
	No	$\sqrt{12}$	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
3	Ina	ivig	IIIB	IVB	VB	VIB	VIIB		- VIII	[IB	IIB	A1	Si	Р	S	Cl	Ar
			3 B	4B	5B	6B	7B				1 B	2B	26.98	28.09	30.97	<u>-</u> 32.07	35.45	39.95
	22.99	24.31	0.1			0.4	0.5		- 8	•	•	20	0.1			2.4	0.5	
4	19 17	$\frac{20}{2}$	21 C	22	23	24 O m	25	26	$\frac{27}{\mathbf{O}}$	28 NT:	29	30	31	32	33	34 C	35	36 17
4	K		\underline{SC}	$\left \frac{11}{12} \right $		\underline{Cr}	<u>Ivin</u>	<u><u>Fe</u></u>		<u>IN1</u>	<u>Cu</u>	<u>Zn</u>	<u>Ga</u>	Ge	<u>As</u>	<u>Se</u>	<u>Br</u>	<u>Kr</u>
	$\frac{39.10}{37}$	40.08	44.96	47.88	50.94 41	52.00 42	54.94 43	55.85 44	58.47 45	58.69 46	63.55 47	65.39 48	69.72 49	72.59	<u>74.92</u> 51	78.96 52	<u>79.90</u> 53	<u>83.80</u> 54
5	Rh	Sr	V	7r	Nh	Mo	Te	Ru	Rh	Pd	Λσ	Cd	In	Sn	Sh	Te	I	Χe
5	<u>10</u> 85 47	87.62	<u> </u>	<u></u>	1 V U 92 91	95 94	$\frac{10}{(98)}$	$\frac{\mathbf{I} \mathbf{u}}{101.1}$	102.9	1064	<u>107</u> 9	$\frac{Cu}{1124}$	114 8	118.7	121.8	$\frac{10}{127.6}$	126 9	$\frac{\Lambda C}{131.3}$
	55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
6	Cs	Ba	La*	Hf	Та	W	Re	Os	Ir	Pt	Au	Ηg	T1	Pb	Bi	Po	At	Rn
	132.9	137.3	138.9	178.5	180.9	183.9	186.2	190.2	190.2	195.1	197.0	200.5	204.4	207.2	209.0	(210)	(210)	(222)
	87	88	89	104	105	106	107	108	109	110	111	112		114		116		118
7	Fr	Ra	<u>Ac</u> ~	<u>Rf</u>	Db	Sg	Bh	Hs	<u>Mt</u>									
	(223)	(226)	(227)	(257)	(260)	(263)	(262)	(265)	(266)	0	0	0		0		0		0

Lanthanide Series*	58 <u>Ce</u> 140.1	59 <u>Pr</u> 140.9	60 <u>Nd</u> 144.2	61 Pm (147)	62 <u>Sm</u> 150.4	$\underbrace{\frac{63}{\underline{Eu}}}_{152.0}$	64 <u>Gd</u> 157.3	65 <u>Tb</u> 158.9	66 Dy 162.5	67 <u>Ho</u> 164.9	68 <u>Er</u> 167.3	69 <u>Tm</u> 168.9	$\frac{70}{\underline{Yb}}_{173.0}$	71 Lu 175.0
Actinide Series~	90 $\underline{Th}_{232.0}$	$\frac{91}{Pa}$	92 $\underbrace{U}_{(238)}$	93 <u>Np</u> (237)	94 <u>Pu</u> (242)	95 Am (243)	96 Cm (247)	97 $\underline{\mathbf{Bk}}_{(247)}$	$98 \\ \underline{Cf}_{(249)}$	99 $\underline{\text{Es}}_{(254)}$	100 Fm (253)	$\frac{101}{\underline{Md}}$	$\frac{102}{No}$ (254)	$\frac{103}{\underline{Lr}}$



Intrinsic Semiconductor

A silicon crystal is different from an <u>insulator</u> because at any temperature above absolute zero temperature, there is a finite probability that an electron in the <u>lattice</u> will be knocked loose from its position, leaving behind an electron deficiency called a "hole".

If a voltage is applied, then both the electron and the hole can contribute to a small <u>current</u> flow.

The conductivity of a semiconductor can be modeled in terms of the <u>band theory</u> of solids. The band model of a semiconductor suggests that at ordinary temperatures there is a finite possibility that electrons can reach the <u>conduction band</u> and contribute to electrical conduction.

The term intrinsic here distinguishes between the properties of pure "intrinsic" silicon and the dramatically different properties of <u>doped</u> <u>n-type</u> or <u>p-type</u> semiconductors.



HyperPhysics**** Condensed Matter

R	Go	Back
Nave		



Semiconductor Current



Further discussion

HyperPhysics**** Condensed Matter

concepts

Go Back

R

Nave



The Doping of Semiconductors

The addition of a small percentage of foreign atoms in the regular crystal lattice of silicon or germanium produces dramatic changes in their electrical properties, producing n-type and p-type semiconductors.





Semiconductor concepts

(3 valence electrons) produce p-type semiconductors by producing a "hole " or electron deficiency.

Gallium

HyperPhysics***** Condensed Matter

Go Back R Nave



Opped Semiconductors





N-Type Semiconductor



•







P-N Junction

One of the crucial keys to <u>solid state electronics</u> is the nature of the P-N junction. When <u>p-type</u> and <u>n-type</u> materials are placed in contact with each other, the junction behaves very differently than either type of material alone. Specifically, current will flow readily in one direction (forward biased) but not in the other (reverse biased), creating the basic <u>diode</u>. This non-reversing behavior arises from the nature of the charge transport process in the two types of materials.



Index

Depletion Region

When a <u>p-n junction</u> is formed, some of the free electrons in the n-region diffuse across the junction and combine with <u>holes</u> to form negative ions. In so doing they leave behind positive ions at the donor <u>impurity</u> sites.





Depletion Region Details



In the p-type region there are holes from the acceptor impurities and in the n-type region there are extra electrons.



When a p-n junction is formed, some of the electrons from the n-region which have reached the conduction band are free to diffuse across the junction and combine with holes.



Electron

Filling a hole makes a negative ion and leaves behind a positive ion on the n-side. A space charge builds up, creating a depletion region which inhibits any further electron transfer unless it is helped by putting a forward bias on the junction.

+)

Positive ion from removal of electron from n-type impurity.

Show effects of biasing.

Negative ion from

filling of p-type

vacancy.

HyperPhysics***** Condensed Matter

Hole

Go Back R Nave

Index

Semiconductor concepts

Semiconductors for electronics



Forward biasing the p-n junction drives holes to the junction from the p-type material and electrons to the junction from the n-type material. At the junction the electrons and holes combine so that a continuous current can be maintained.

Diodes



Index





$\Theta \Theta \Theta$

O Transistors

The Junction Transistor



Transistor as Current Amplifier

The larger <u>collector current</u> ${}^{I}c$ is proportional to the base current ${}^{I}B$ according to the relationship ${}^{I}c = \beta {}^{I}B$, or more precisely it is proportional to the base-emitter voltage ${}^{V}BE$. The smaller base current controls the larger collector current, achieving current amplification.



Index

Semiconductor

 $\Theta \Theta \Theta$





Transistor Switch Example

The switch is closed.

Open the switch

The base resistor is chosen small enough so that the base current drives the transistor into <u>saturation</u>.

In this example the mechanical switch is used to produce the base current to close the transistor switch to show the principles. In practice, any voltage on the base sufficient to drive the transistor to saturation will close the switch and light the bulb.



Transistor Switches



from: http://ece-www.colorado.edu/~bart/book/mosintro.htm



Enhancement-mode (Normally-off) MOSFET N-channel

Vg < Vt: gate bias is less positive than the threshold voltage. Not enough electrons and no inversion channel is formed.





Enhancement-mode (Normally-off) MOSFET N-channel Vg > Vt: gate bias is more positive than the threshold voltage. Sufficient electrons accumulate and forms the inversion channel.



Applet started.

http://jas2.eng.buffalo.edu/applets/education/mos/mosfet/mos

An Inverter using MOSFET

- CMOS = complementary metal oxide semiconductor
- P-type transistor conducts when gate is low
- N-type transistor conducts when gate is high





NAND GATE



Z









NOR GATE

A	В	Z		
0	0	1		
0	1	0		
1	0	0		
1	1	0		



CMOS Logic vs Bipolar Logic

- MOSFET transistors are easier to miniaturize
- CMOS logic has lower current drain
- CMOS logic is easier to manufacture
References

• Materials on semiconductors, PN junction and transistors taken from the HyperPhysics web site:

<http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html>