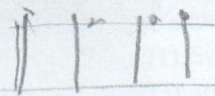


## 194 Fast Fourier Transform

as in Exercise 6.1. The twiddle constants are the same as in Exercise 6.1. Note that the twiddle constant  $W$  is multiplied with the second term only (not with the first).



Stage 1

$$x(0) = \dots x(3) = 1$$

$$x(4) = \dots x(7) = 0$$

$N=8$ , Four Twiddles.

$$W^0 = 1$$

$$W^1 = e^{-j2\pi/8} = e^{-j\pi/4} = -j$$

$$W^2 = e^{-j4\pi/8} = -1$$

$$W^3 = e^{-j6\pi/8} = -j$$

where the sequence  $x$ 's represents the intermediate output after the first iteration and becomes the input to the subsequent stage.

Stage 2

$$\begin{aligned} x'(0) + W^0 x'(2) &= 1 + 1 = 2 \rightarrow x''(0) \\ x'(4) + W^2 x'(6) &= 1 + (-1) = 0 \rightarrow x''(4) \\ x'(0) - W^0 x'(2) &= 1 - 1 = 0 \rightarrow x''(2) \\ x'(4) - W^2 x'(6) &= 1 - (-1) = 2 \rightarrow x''(6) \\ x'(1) + W^0 x'(3) &= 1 + 1 = 2 \rightarrow x''(1) \\ x'(5) + W^2 x'(7) &= 1 + (-1) = 0 \rightarrow x''(5) \\ x'(1) - W^0 x'(3) &= 1 - 1 = 0 \rightarrow x''(3) \\ x'(5) - W^2 x'(7) &= 1 - (-1) = 2 \rightarrow x''(7) \end{aligned}$$

where the intermediate second-stage output sequence  $x''$ 's becomes the input sequence to the final stage.

Stage 3

$$\begin{aligned} X(0) &= x''(0) + W^0 x''(1) = 4 \\ X(1) &= x''(4) + W^1 x''(5) = 1 - j2.414 \\ X(2) &= x''(2) + W^2 x''(3) = 0 \\ X(3) &= x''(6) + W^3 x''(7) = 1 - j0.414 \\ X(4) &= x''(0) - W^0 x''(1) = 0 \\ X(5) &= x''(4) - W^1 x''(5) = 1 + j0.414 \\ X(6) &= x''(2) - W^2 x''(3) = 0 \\ X(7) &= x''(6) - W^3 x''(7) = 1 + j2.414 \end{aligned}$$

which is the same output sequence as found in Exercise 6.1.

## 6.5 BIT

A bit-reversed sequence is a sequence where the bits of the address are reversed. For example, the address 1011 (decimal 11) is reversed to 1101 (decimal 13). This bit-reversal is done for  $N=8$  and the twiddle constants are the same as in Exercise 6.1.

## 6.6 DEV

A radix-4 decimation-in-time FFT algorithm is a fast Fourier transform algorithm that uses a radix-4 butterfly structure. It is a four-stage algorithm that uses a radix-4 butterfly structure to compute the FFT. The input sequence is bit-reversed and the output sequence is in natural order.

$X(0)$

Let  $n = n$   
tions, resp

$X$

which represents