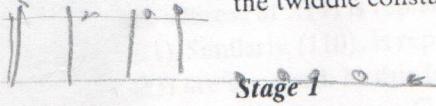


as in Exercise 6.1. The twiddle constants are the same as in Exercise 6.1. Note that the twiddle constant W is multiplied with the second term only (not with the first).



Stage 1

$$x(0) = \dots x(3) = 1$$

$$x(4) = \dots x(7) = 0$$

$N = 8$, Four Twiddles.

$$w^0 = 1$$

$$w^1 = e^{-j\frac{2\pi}{8}} = 0.707 - j0.707$$

$$w^2 = e^{-j\frac{4\pi}{8}} = -j$$

$$w^3 = e^{-j\frac{6\pi}{8}} = -0.707 - j0.707$$

$$\left. \begin{aligned} x(0) + W^0 x(4) &= 1 + 0 = 1 \rightarrow x'(0) \\ x(0) - W^0 x(4) &= 1 - 0 = 1 \rightarrow x'(4) \\ x(2) + W^0 x(6) &= 1 + 0 = 1 \rightarrow x'(2) \\ x(2) - W^0 x(6) &= 1 - 0 = 1 \rightarrow x'(6) \\ x(1) + W^0 x(5) &= 1 + 0 = 1 \rightarrow x'(1) \\ x(1) - W^0 x(5) &= 1 - 0 = 1 \rightarrow x'(5) \\ x(3) + W^0 x(7) &= 1 + 0 = 1 \rightarrow x'(3) \\ x(3) - W^0 x(7) &= 1 - 0 = 1 \rightarrow x'(7) \end{aligned} \right\}$$

$$w^0 = e^{-j\frac{2\pi}{N}}$$

$$w^1 = e^{-j\frac{2\pi}{8}} = 0.707 - j0.707$$

$$w^2 = e^{-j\frac{4\pi}{8}} = -j$$

$$w^3 = e^{-j\frac{6\pi}{8}} = -0.707 - j0.707$$

$$w^4 = e^{-j\frac{8\pi}{8}} = -1 - 0$$

$$w^5 = e^{-j\frac{10\pi}{8}} = -0.707 + j0.707$$

$$w^6 = e^{-j\frac{12\pi}{8}} = -0.707 + j0.707$$

$$w^7 = e^{-j\frac{14\pi}{8}} = 0.707 + j0.707$$

where the sequence x' 's represents the intermediate output after the first iteration and becomes the input to the subsequent stage.

Stage 2

$$\left. \begin{aligned} x'(0) + W^0 x'(2) &= 1 + 1 = 2 \rightarrow x''(0) \\ x'(4) + W^2 x'(6) &= 1 + (-j) = 1 - j \rightarrow x''(4) \\ x'(0) - W^0 x'(2) &= 1 - 1 = 0 \rightarrow x''(2) \\ x'(4) - W^2 x'(6) &= 1 - (-j) = 1 + j \rightarrow x''(6) \\ x'(1) + W^0 x'(3) &= 1 + 1 = 2 \rightarrow x''(1) \\ x'(5) + W^2 x'(7) &= 1 + (-j)(1) = 1 - j \rightarrow x''(5) \\ x'(1) - W^0 x'(3) &= 1 - 1 = 0 \rightarrow x''(3) \\ x'(5) - W^2 x'(7) &= 1 - (-j)(1) = 1 + j \rightarrow x''(7) \end{aligned} \right\}$$

where the intermediate second-stage output sequence x'' 's becomes the input sequence to the final stage.

6.5 BIT

A bit-rev this bit-s are swap the addr $X(1)$. Sin $X(3)$ are for the in This b for $N = 6$ and the t Several terms fo

6.6 DEV

A radix-4 grams on in-freque radix-4 Fl inputs and of stages four stage summatio

$X($

Let $n = n$ sons, resp

Stage 3

$$\begin{aligned} x''(0) &= x'(0) + W^0 x'(2) + W^0 [x'(1) + W^0 x'(3)] \\ &= x(0) + W^0 x(4) + W^0 [x(2) + W^0 x(6)] \\ &\quad + W^0 [x(1) + W^0 x(5) + W^0 [x(3) + W^0 x(7)]] \\ &= x(0) + W^0 x(4) + W^0 x(2) + W^0 x(6) \\ &\quad + W^0 x(1) + W^0 x(5) + W^0 x(3) + W^0 x(7) \\ &= x(0) + W^0 x(1) + W^0 x(2) + W^0 x(3) + W^0 x(4) \\ &\quad + W^0 x(5) + W^0 x(6) + W^0 x(7) \end{aligned}$$

$$x'(0) + W^0 x'(2) =$$

$$\begin{aligned} X(0) &= x''(0) + W^0 x''(1) = 4 \\ X(1) &= x''(4) + W^1 x''(5) = 1 - j2.414 \\ X(2) &= x''(2) + W^2 x''(3) = 0 \\ X(3) &= x''(6) + W^3 x''(7) = 1 - j0.414 \\ X(4) &= x''(0) - W^0 x''(1) = 0 \\ X(5) &= x''(4) - W^1 x''(5) = 1 + j0.414 \\ X(6) &= x''(2) - W^2 x''(3) = 0 \\ X(7) &= x''(6) - W^3 x''(7) = 1 + j2.414 \end{aligned}$$

which is the same output sequence as found in Exercise 6.1.

$$x(0) = \sum_{n=0}^{N-1} x(n) W^{nk} = x(0) W^0 + x(1) W^0 + x(2) W^0 + x(3) W^0 + \dots x(7) W^0$$