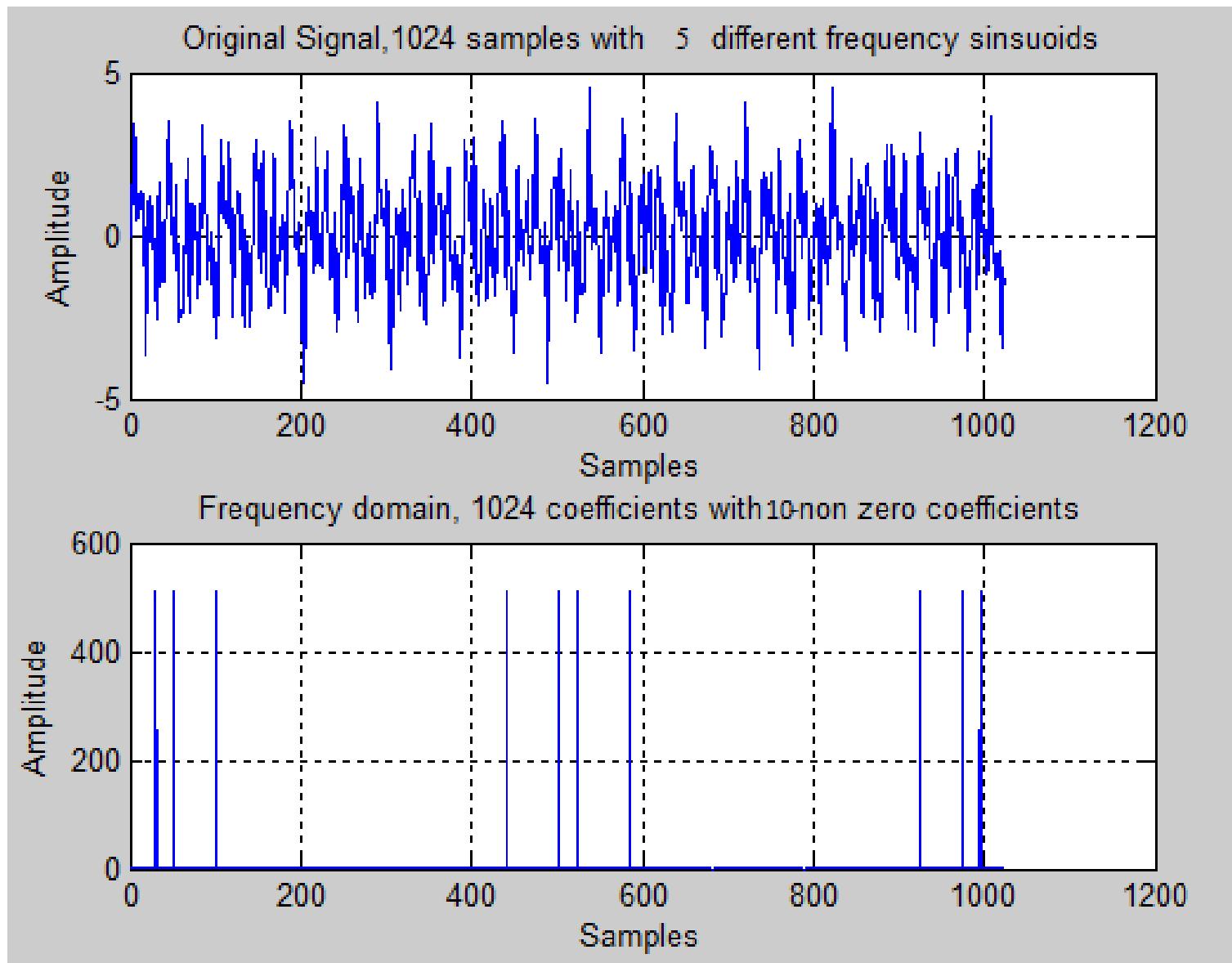


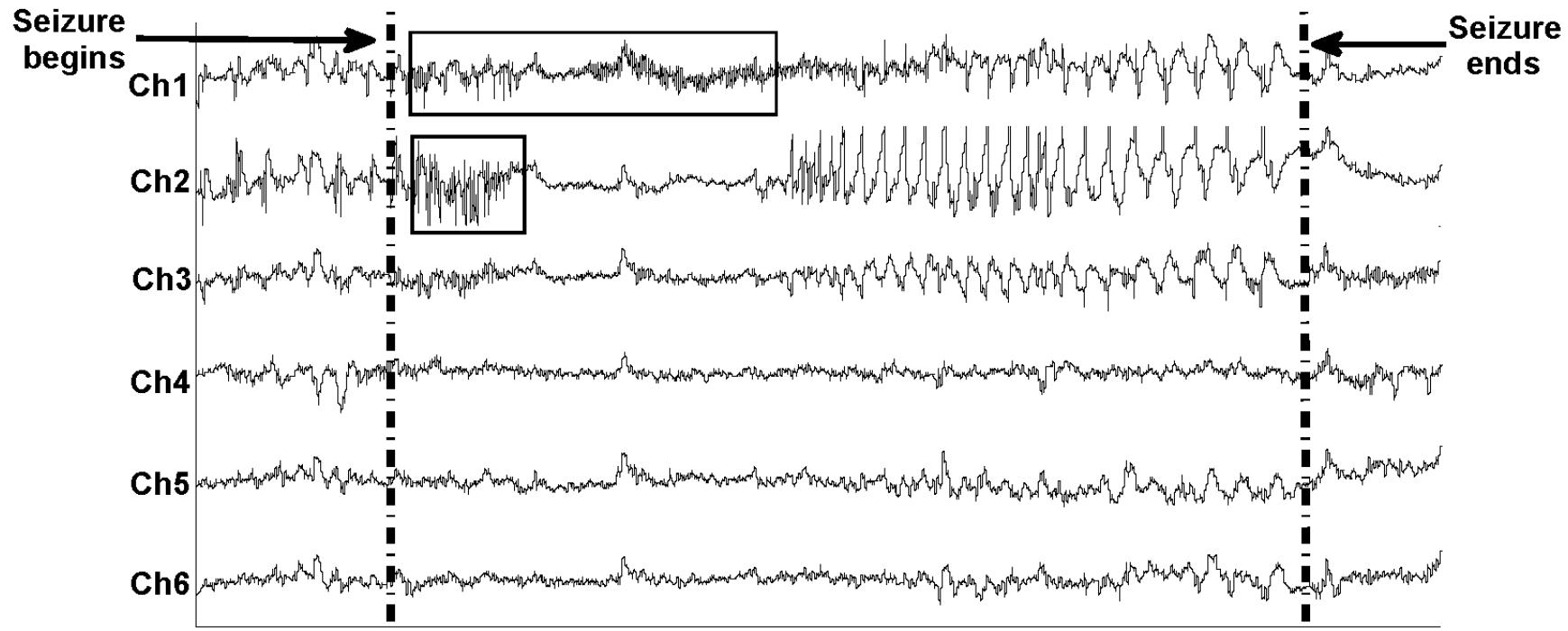
FFT Application Examples and Implementation

FFT Example 1: Signal Sparsity in time Frequency Domain



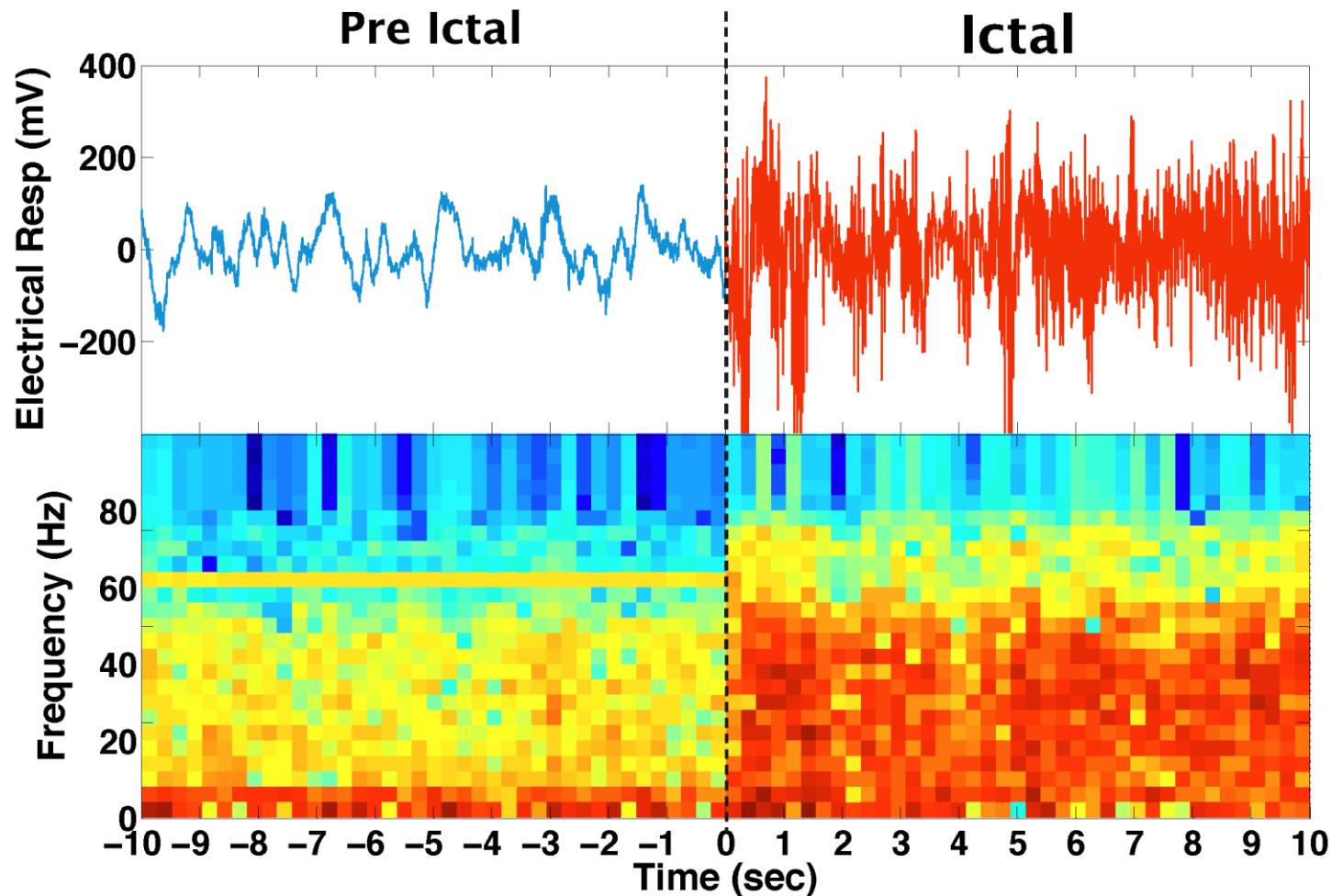
FFT Example 2: Seizure Detection Problem

- Electrical signals can be detected by EEG signals before or just at the start of clinical symptoms
 - The ability to detect can be used to warn the patient or alert caregiver



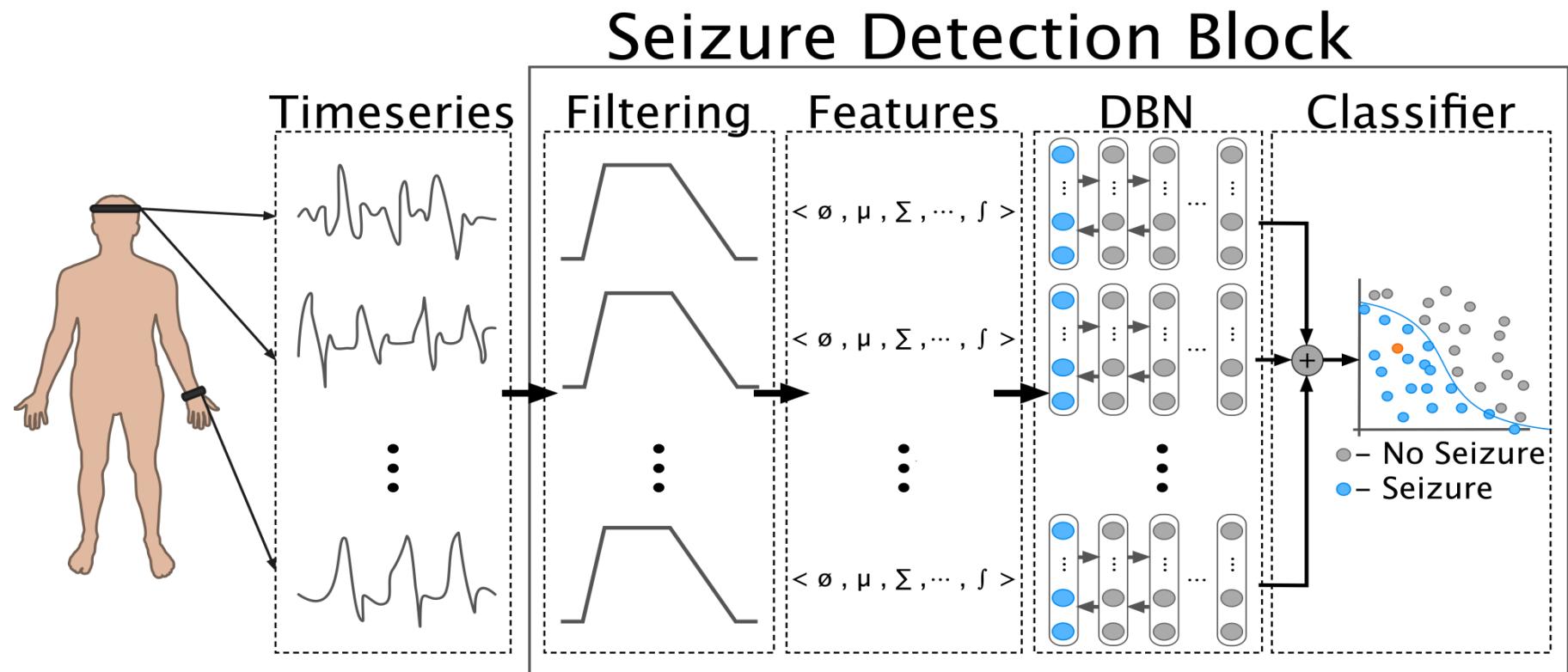
FFT Example 2: For Seizure, Frequency and Time domain analysis

- “We [doctors] detect seizures by looking at the evolution of frequency and amplitude in EEG” Dr. Jennifer Hopp, Epilepsy Center, UMMC.



A wearable solution for Multi-physiological signal processing

- Use FFT and filtering



FFT N-point implementation from FFT-2

• FFT $\cdot N$ where N is Power of 2 .

— First calculate the Twiddle factors for half of N ($\frac{N}{2}$) →

— Construct the diagram of FFT N

— Construct a 2-point FFT

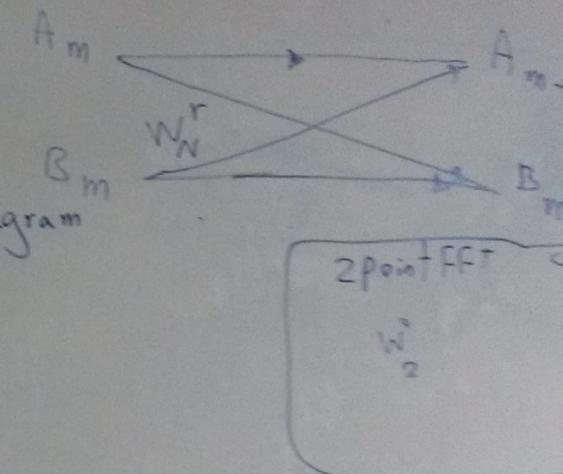
— Build the FFT N using the 2-point FFT and diagram

→ 1) There are $\log N$ stages

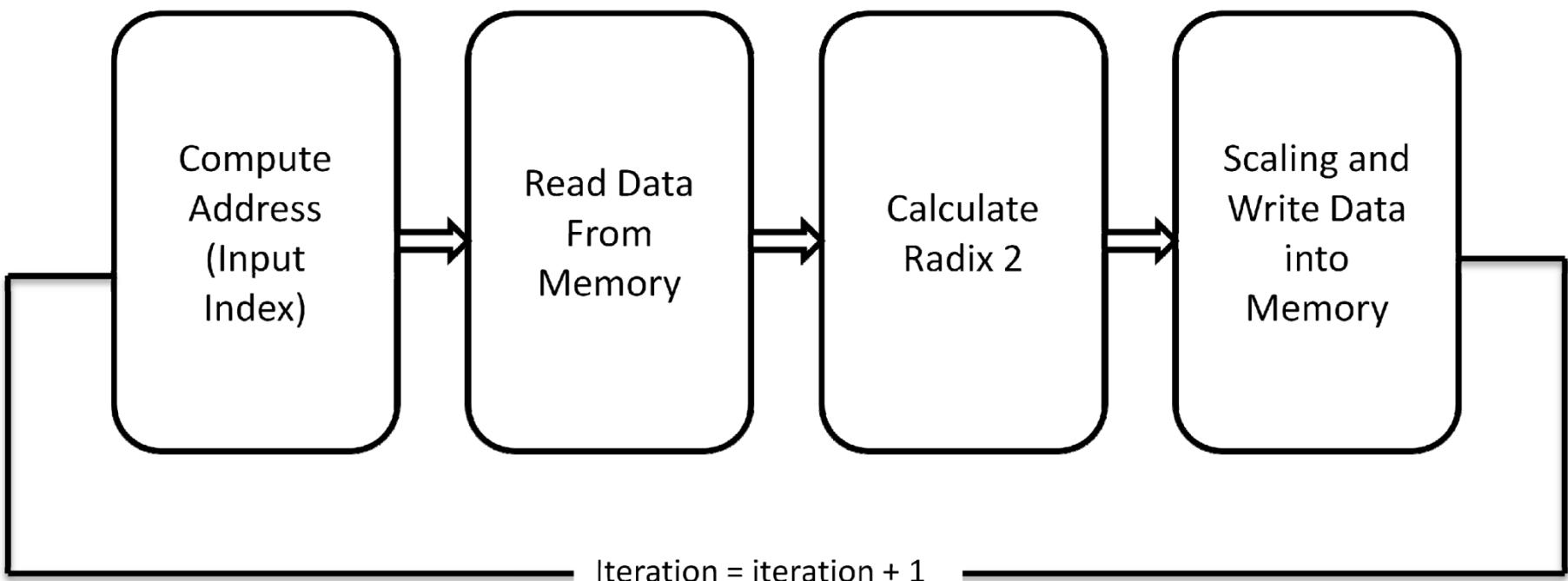
2) There are $\frac{N}{2}$ of 2-point FFT per stage.

3) The orders of inputs are in bit reverse

4) For Twiddle factors: It shuffles between $2^{(\text{stage}-1)}$ samples, where $\text{stage} = \log N$

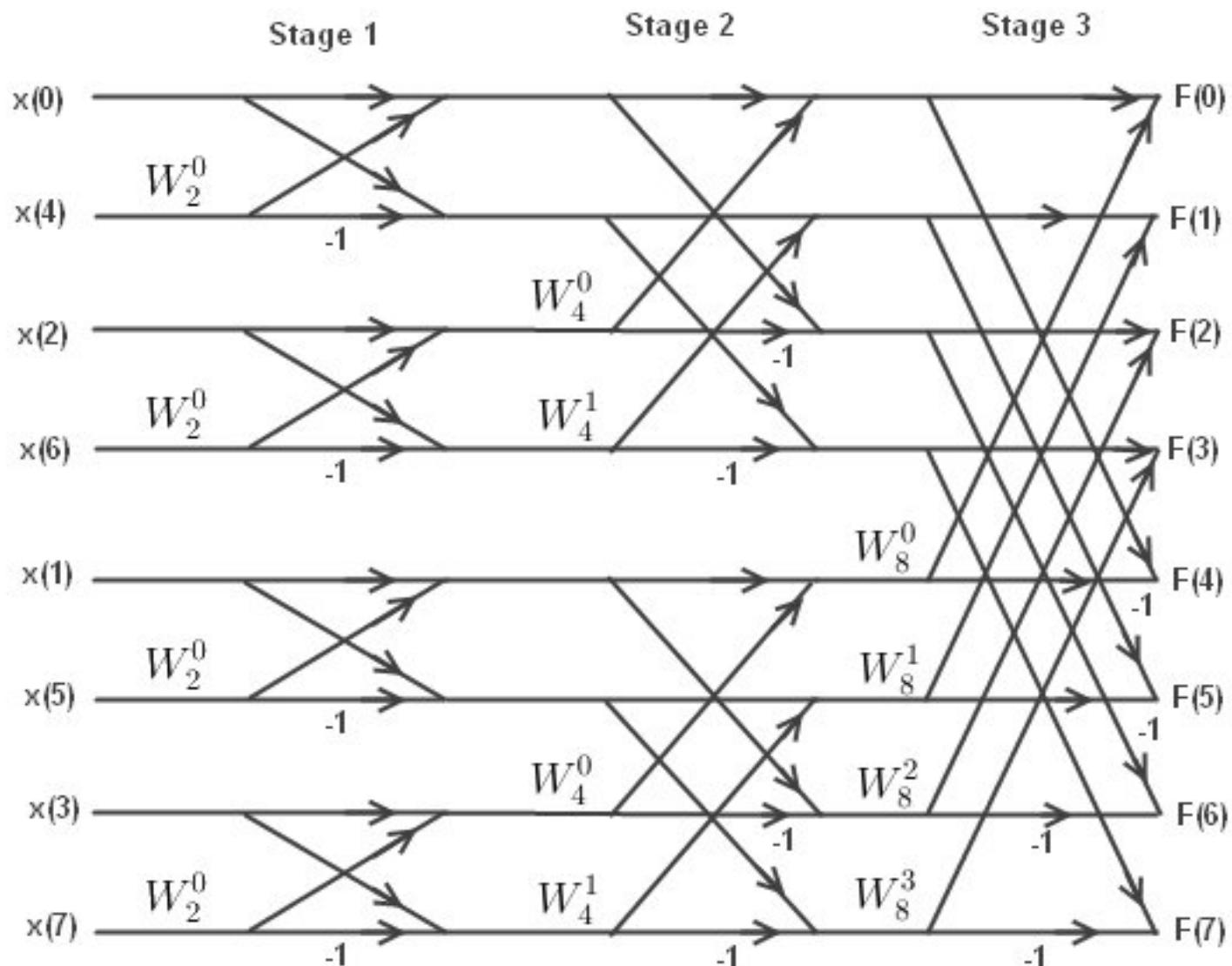


FFT-N Calculation from FFT-2 (Radix-2)

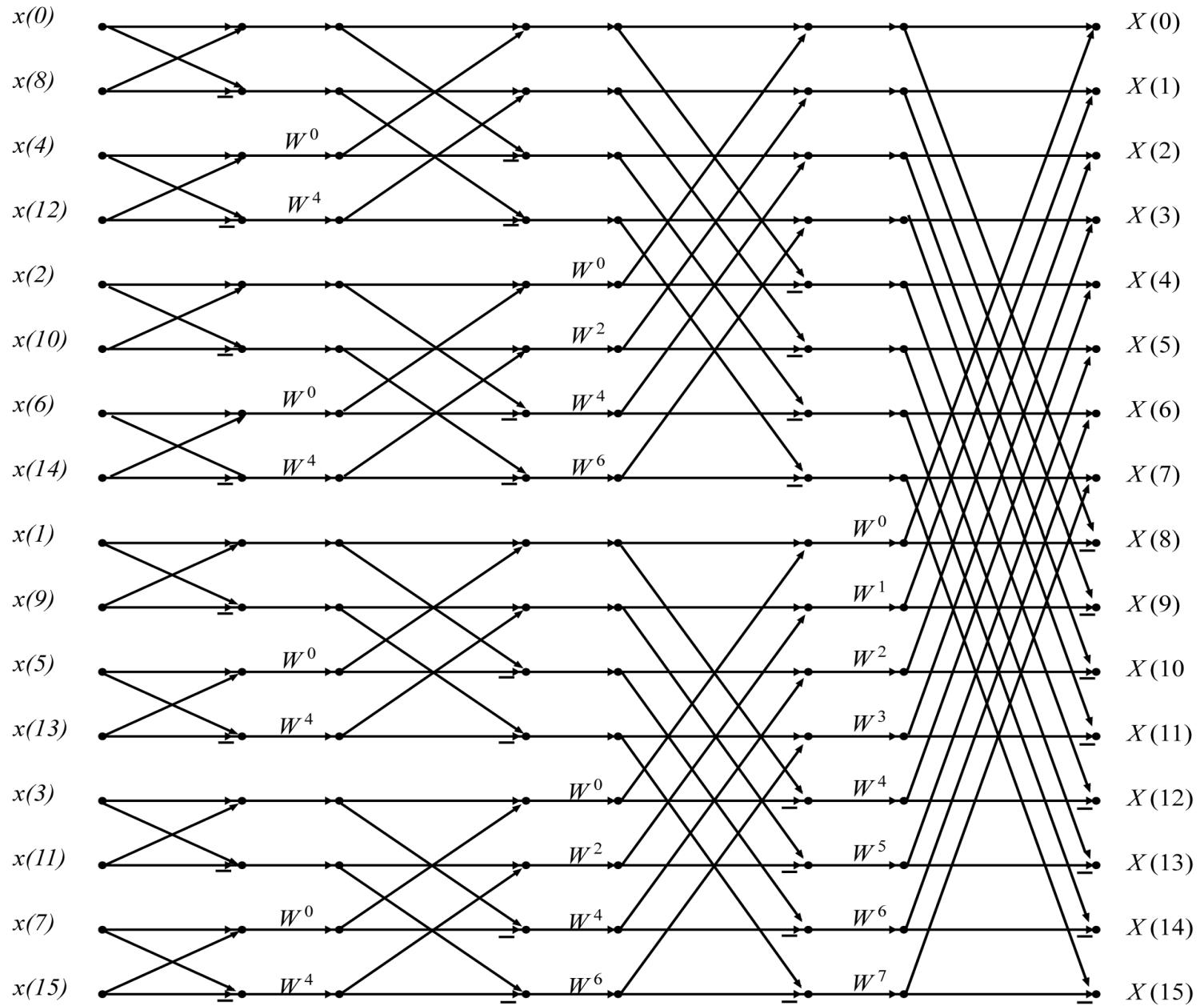


FFT 8-Point

An 8 Input Butterfly. Note, you double a 4 input butterfly, extend output lines, then connect the upper and lower butterflies together with diagonal lines.



FFT 16-Point



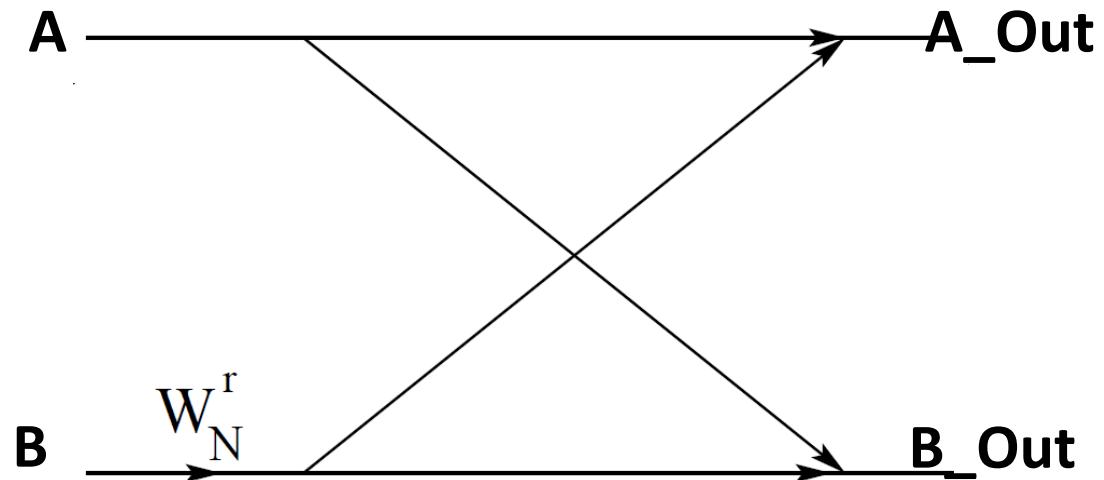
Twiddle factor Computation in Matlab

- Twiddle factor calculation in Matlab:
- Ex: N=8,
 - $W_0 = \exp(-2\pi \cdot 0 \cdot i/N) = 1$
 - $w_1 = \exp(-2\pi \cdot 1 \cdot i/N)$
 -
 - $W_7 = \exp(-2\pi \cdot 7 \cdot i/N)$

Radix-2 Butterfly Implementation

$$A_Out = A + W * B$$

$$B_Out = A - W * B$$



Where $A = Ar + iAi$, $B = Br + iBi$, $W = Wr + iWi$

$$\text{So, } A_{out} = (Ar + WrBr - WiBi) + (Br + WrBi + WiBr);$$

$$B_{out} = (Ar - WrBr + WiBi) + (Br - WrBi - WiBi);$$

FFT inputs bit reversal and memory addressing

- Bit reversal inputs example to FFT 8-point diagram

0 = 000 reversed \rightarrow 000 = 0

1 = 001 reversed \rightarrow 100 = 4

2 = 010 reversed \rightarrow 010 = 2

3 = 011 reversed \rightarrow 110 = 6

4 = 100 reversed \rightarrow 001 = 1

5 = 101 reversed \rightarrow 101 = 5

6 = 110 reversed \rightarrow 011 = 3

- Refer to for FFT-8 example (very useful) in slides:
http://www.csee.umbc.edu/~tinoosh/cmpe691/slides/osattari_ms.pdf

FFT Memory storage requirements for serial implementation

Memory Requirement.

①

Block RAM: IF overwriting Per Stage

For $\underline{\underline{FFT}}_N$, you need $\underline{\underline{N}}$ entries for Memory

For 16-bit Real and Imag $\Rightarrow N \times 32$ bit. Mem

$$\lg \frac{16}{2} = 4 \text{ stages}$$

$$\text{Per stage} = \frac{16}{2} = 8$$

FFT Twiddle factor Memory

Memory Requirement

(2) ROM/LUT for W factors

For FFT N: $e^{\frac{-j2\pi k}{N}}$ \rightarrow k is $0 \dots \frac{N}{2}$ $\frac{N}{2}$ W factor

- No. of entries:

FFT N: we need $\frac{N}{2}$ of real + $\frac{N}{2}$ of imag numbers.

- For W integer bits: $-1 < w < 1 \Rightarrow$ 2 bits inter-

More details for FFT

- For more details refer to these documents:
- http://www.csee.umbc.edu/~tinoosh/cmpe691/slides/osattari_ms.pdf
- <http://www.csee.umbc.edu/~tinoosh/cmpe691/slides/Handout.fft1.diagrams.pdf>
- And FFT example from the book in the slides