3.1 Coordinate Systems

3.1.1 Rectangular Coordinates

The coordinate system is illustrated in Figure 3.1. The location of a point in three dimensional space may be specified by an ordered set of numbers (x, y, z). The ranges for the coordinate parameters are:

$$\begin{aligned} -\infty \leq x \leq \infty \\ -\infty \leq y \leq \infty \\ -\infty \leq z \leq \infty \end{aligned} \tag{3.1}$$

The unit vectors $\hat{\mathbf{x}} \equiv \underline{\delta}_{\mathbf{x}}$, $\hat{\mathbf{y}} \equiv \underline{\delta}_{\mathbf{y}}$, and $\hat{\mathbf{z}} \equiv \underline{\delta}_{\mathbf{z}}$ are also illustrated in Figure

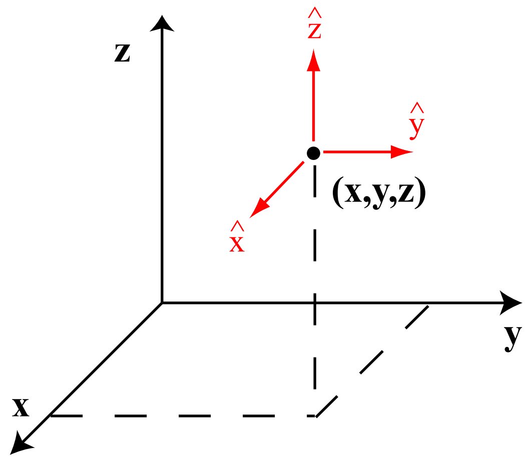

Figure 3.1: The rectangular coordinate system.

3.1. These unit vectors are every where mutually orthogonal. The "del" operator in rectangular coordinates is simply:

$$\underline{\nabla} = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z} \ . \tag{3.2}$$

The Laplacian operator in rectangular coordinates is :

$$\underline{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \ . \tag{3.3}$$

3.1.2 Cylindrical Coordinates

The coordinate system is illustrated in Figure 3.2. The location of a point in three dimensional space may be specified by an ordered set of numbers (r, θ, z). The ranges for the coordinate parameters are:

$$\begin{aligned} 0 \leq r &\leq \infty \\ 0 \leq \theta &\leq 2\pi \\ -\infty \leq z &\leq \infty \end{aligned} \tag{3.4}$$

The relationship between rectangular and cylindrical coordinates is summarized as follows:

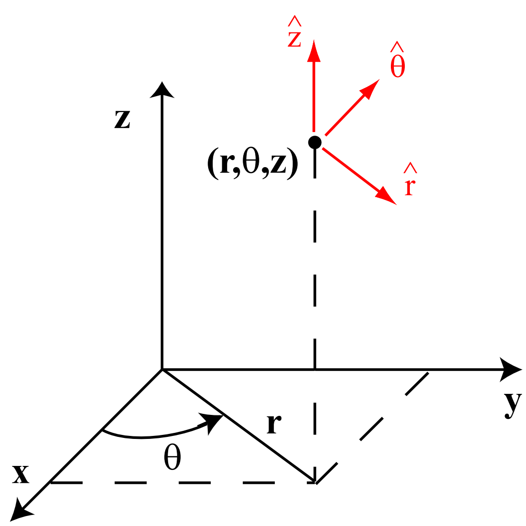

Figure 3.2: The cylindrical coordinate system.

$$\begin{aligned} x &= r\cos\theta & r &= +\sqrt{x^2+y^2} \\ y &= r\sin\theta & \theta &= \arctan(y/x) \\ z &= z & z &= z \end{aligned}$$

The unit vectors $\hat{\mathbf{r}} \equiv \underline{\delta}_{\mathbf{r}}$, $\hat{\theta} \equiv \underline{\delta}_{\theta}$, and $\hat{\mathbf{z}} \equiv \underline{\delta}_{\mathbf{z}}$ are also illustrated in Figure 3.2. These unit vectors are every where mutually orthogonal. In contrast to rectangular coordinates, the unit vectors $\hat{\mathbf{r}}$ and $\hat{\theta}$ change direction depending on the particular point in space. For this reason, it is critical to take care when executing differential operations in cylindrical coordinates. For example, $\frac{\partial}{\partial\theta}\hat{\mathbf{r}} = \hat{\theta}$ and $\frac{\partial}{\partial\theta}\hat{\theta} = -\hat{\mathbf{r}}$. The "del" operator in cylindrical coordinates is:

$$\underline{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\mathbf{z}}\frac{\partial}{\partial z} . \tag{3.5}$$

The Laplacian operator in cylindrical coordinates is :

$$\underline{\nabla}^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2} . \tag{3.6}$$

3.1.3 Spherical Coordinates

The coordinate system is illustrated in Figure 3.3. The location of a point in three dimensional space may be specified by an ordered set of numbers (r, θ, ϕ). The ranges for the coordinate parameters are:

$$\begin{aligned} 0 &\le r \le \infty \\ 0 &\le \theta \le \pi \\ 0 &\le \phi \le 2\pi \end{aligned} \tag{3.7}$$

Note carefully that the definitions of θ are very different in the cylindrical and spherical coordinate systems! The relationship between rectangular and spherical coordinates is summarized as follows:

$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= +\sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \arctan\frac{\sqrt{x^2+y^2}}{z} \\ z &= r\cos\theta & \phi &= \arctan(y/x) \end{aligned}$$

The unit vectors $\hat{\mathbf{r}} \equiv \underline{\delta}_{\mathbf{r}}$, $\hat{\theta} \equiv \underline{\delta}_{\theta}$, and $\hat{\phi} \equiv \underline{\delta}_{\phi}$ are also illustrated in Figure 3.3. These unit vectors are every where mutually orthogonal. In contrast to rectangular coordinates, each of these unit vectors changes direction depending on the particular point in space. For this reason, it is critical to take care when executing differential operations in spherical coordinates. For example, $\frac{\partial}{\partial\phi}\hat{\mathbf{r}} = \hat{\phi}\sin\theta$ and $\frac{\partial}{\partial\phi} = -\hat{\mathbf{r}}\sin\theta - \hat{\theta}\cos\theta$. The "del" operator in spherical coordinates is:

$$\underline{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} . \tag{3.8}$$

The Laplacian operator in spherical coordinates is :

$$\underline{\nabla}^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2} . \tag{3.9}$$

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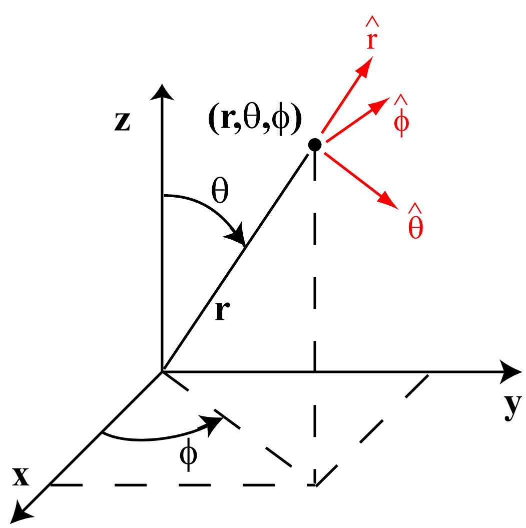

Figure 3.3: The spherical coordinate system.