Non-Uniform Time-Stepping For Fast Simulation of Photodetectors Under High-Peak-Power, Ultra-Short Optical Pulses

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***Abstract*—A novel non-uniform time-stepping procedure is developed to reduce the memory usage and simulation time—by two orders of magnitude—of photodetectors when detecting high-peak-power, ultra-short optical pulses. The proposed procedure can be used in other marching-on-in-time solvers to achieve the same for the simulations dealing with ultra-short pulses.**

***Index Terms*—photodetectors, photodiodes, semiconductor device modeling, non-uniform time-stepping.**

I. Introduction

High-current, high-power photodetectors are essential elements in RF-photonics, optical communications, photonic microwave generation, and high-frequency measurement. Numerical methods are widely used to study their nonlinearity, to determine the sources of noise and distortion, and to design photodetectors meeting the specific requirements of a given application. The calculation of the impulse response is a critical building block in numerical studies. From the impulse response, it is possible to infer key performance metrics such as the phase noise and responsivity. Furthermore, the convolution of the impulse response with the input data signal yields the eye diagram of the photoreceiver, which is a key element of performance assessment at the system level and link-budget determination.

A robust and more efficient approach compared to Monte Carlo simulations [1] to calculate the impulse response is to solve the drift-diffusion equations [2], [3] in a marching-on-in-time scheme implemented with sufficiently small time steps to prevent convergence issues. However, choosing very small time steps increases the overall computation time and memory usage. For the simulations of photodetectors for detecting high-peak-power, ultra-short optical pulses, this ultra-fine time-stepping might cause out-of-memory problems by requiring a memory allocation exceeding the physical memory capacity of a personal computer. In this work, we propose a novel non-uniform time-stepping (TS) approach that reduces the memory requirements and impulse response computation time by two orders of magnitude. The proposed procedure can be used in other marching-on-in-time solvers to achieve the same.

II. Impulse Response Calculation and Time-Stepping Methods

In [2] and [3], a drift-diffusion model is used that includes impact ionization, thermionic field emission, the Franz-Keldysh effect, and bleaching to study the modified uni-traveling carrier (MUTC) photodetector that was originally proposed in [4]. To calculate the impulse response of the device under pulsed excitations, first, the dark current is calculated. Then, the structure is excited with an ultra-short pulse defined by $y(t) = A\text{sech}\{(t - t_c)/\tau\}$ where τ, t_c, and A are the optical pulse duration, pulse position, and pulse amplitude, respectively, and the output current, I_{out}, is calculated as a function of time. The normalized impulse response, $h(t) = \Delta I_{\text{out}}(t)/\int_0^\infty \Delta I_{\text{out}}(t)\text{d}t$ is then calculated.

In [2] and [3], the authors computed this integral using a uniform TS, i.e. $t_{\text{lin}}[i] = \Delta t \times (i - 1)$, where t_R is the pulse repetition period, $\Delta t = t_R/(N - 1)$, N is the total number of the time steps, and $i = 1, 2, \cdots, N$. Again in [2], [3], the impulse response is plotted in a log-lin scale to clearly show rapidly changing behavior around the pulse center. Based on this observation, a logarithmic TS was recently proposed to evaluate this numerical integration more efficiently [5], where the number of the time steps can be chosen much smaller than N. In this approach, $t_{\log,B}$, the time steps logarithmically distributed between t_c and t_R is determined using

$$t_{\log,B}[i] = t_R^{(i-1)/(M-1)} t_c^{(M-i)/(M-1)}, \tag{1}$$

for $i = 1, 2, \cdots, M$. Then, a similar formula is used to determine $t_{\log,A}$, the time steps from 0 to t_c, excluding t_c, i.e. $t_{\log} = t_{\log,A} \cup t_{\log,B}$ as shown in Fig. 1. By doing so, $\Delta t[i]$, the difference between consecutive time steps, is decreased around t_c and increased as one moves away from t_c. With this approach, a factor of 20 reduction in computation time is achieved for the analysis of the photodetector under continuous excitations [5], in which the excitation is perturbed with $y(t)$ assuming a small A. However, in an MUTC that is detecting high-peak-power, ultra-short optical pulses, the logarithmic TS does not provide a sufficient number of points around the pulse to capture the rapidly changing fields and currents along the photodetector.

In order to address this issue, we propose a novel non-uniform TS by even further decreasing the $\Delta t[i]$ around the

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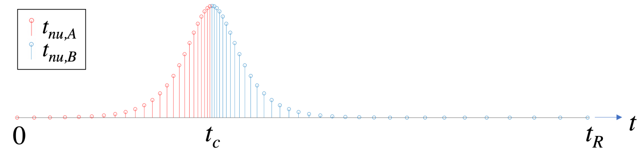

Fig. 1. Schematic illustration of non-uniform time-stepping, where the time steps logarithmically distributed between t_c and t_R and between t_c and $t = 0$ are determined separately.

pulse center and increasing $\Delta t[i]$ as $|t - t_c|$ increases, with the expression of

$$t_{\mathrm{nu},B}[i] = t_c + \left(\frac{t_R - t_c}{\sum_{k=1}^{M} \xi(k)}\right) \sum_{k=1}^{i} \xi(k), \tag{2}$$

where

$$\xi(k) = \{t_{\log,B}[k] - t_{\log,B}[k-1]\} \left(\frac{k + c_1}{M + c_1}\right)^{c_2}, \tag{3}$$

and c_1 and c_2 are coefficients that we can choose to control the amount we shrink/enlarge $\Delta t[i]$. Note that for the non-uniform TS, we define $t_{\log,B}[0] = t_{\log,B}[1]$, so that $t_{\mathrm{nu},B}[1] = t_c$. This approach greatly reduces the number of time steps needed without sacrificing accuracy.

III. Results and Discussions

Figure 2(a) compares the computation times of the uniform, logarithmic, and non-uniform TS implementations as a function of average incident power for the MUTC photodetector described in [3] for detecting 100-fs optical pulses with a 50-MHz repetition rate. For the logarithmic TS, $M = N/a_F$, where a_f is the approximate acceleration factor, which is set to 100. For the non-uniform TS implementation, $c_1 = M/10$ and $c_2 = 2$. Numerical results show that as the incident power increases from microwatts to milliwatts, the computation time increases from a few hours to hundreds of hours in the uniform TS implementation. Both logarithmic and non-uniform TS implementations require much less computation time but the former is not able to proceed above 10 mW for $N = 400,000$ and $a_F = 100$. However, even under 100 mW ultra-short pulses, the non-uniform TS implementation completes the calculations in about 10 hours on a desktop computer with a 3.1 GHz processor without any parallelization. The average Δt is 0.15 fs around the pulse center, which would require 6 million time steps in the uniform implementation to guarantee a similar accuracy and this simulation would take approximately 6 weeks on the same desktop computer used in this study.

Figure 2(b) follows the notation used in Fig. 2(a) for the memory usage, which is computed by taking all the arrays into the account used during impulse response calculations. Again, we observe a two-orders-of-magnitude reduction in memory usage. However, it should be noted that the minimum number of time steps required for accurate enough solutions can be lower than the ones that we actually used. Hence, the slopes of the curves presented in Figs. 2(a) and (b) could be slightly different when the number of time steps is changed.

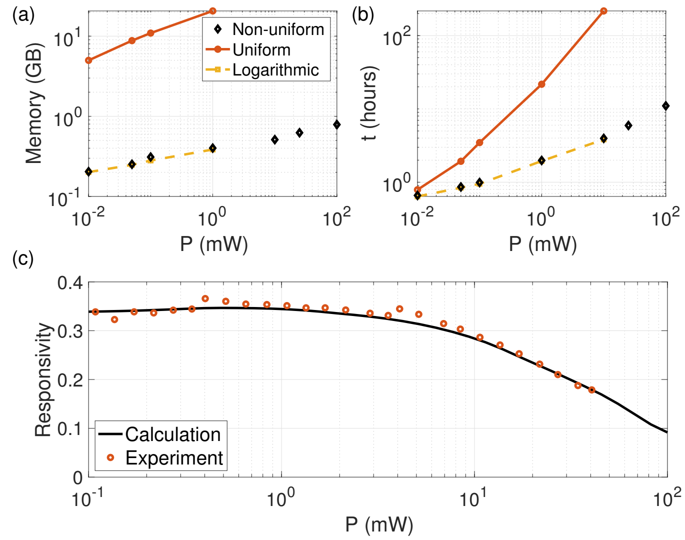

Fig. 2. (a) Computation time and (b) memory usage of the uniform, logarithmic, and non-uniform TS implementations as a function of the average incident power. Neither the linear nor the logarithmic TS is able to proceed above 10 mW for $N = 400,000$ and $a_F = 100$. (c) Responsivity as a function of average incident power, where the black solid curve and red circles represent numerical and experimental [3] results, respectively. Numerical results assume $A = C = 1$, $B = 10^{-3}$, $D = 0.05$, and $E = 10^{-4}$ for the bleaching coefficients, which differs from the coefficients that are used in [3].

Figure 2(c) demonstrates good agreement between the experimental and numerical results for the responsivity of the MUTC photodetector as a function of average incident power.

IV. Conclusion

In conclusion, the proposed non-uniform time-stepping procedure enables a two-orders-of-magnitude reduction in the memory usage and simulation time to determine the impulse response of an MUTC photodetector that detects high-peak-power, ultra-short optical pulses. This approach will be useful for any photodetector that is excited with high-energy optical pulses. It can also be used in other marching-on-in-time solvers.

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