Formation stress effects on wave propagation in a fluid-filled borehole

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Abstract— **Formation stress magnitudes provide useful input to design decisions in wellbore planning, wellbore stability, and reservoir management in the oil and gas industry. It has been known for the past 50 years that elastic wave velocities in rocks change as a function of applied stress. Yet reliable inversion techniques are not available for estimating formation stresses from measured changes in sonic velocities. Borehole wave propagation in such formations can be described by equations of motion for small dynamic fields superposed on a bias. These equations are derived from the rotationally invariant equations of nonlinear elasticity by making a Taylor expansion of the quantities for the dynamic state about their values in the biasing (or intermediate) state. The effective elastic constants in these equations become position-dependent in the presence of inhomogeneous stresses. These equations can be solved either by a finite-difference or perturbation techniques. A finite-difference formulation of equations of motion in the presence of such stresses yields a complete wave solution produced by either a monopole or dipole band-limited source placed in a fluid-filled borehole. Processing of synthetic waveforms by a modified matrix pencil algorithm isolates various dispersive and non-dispersive arrivals. While a perturbation method is an expedient way of solving equations of motion with spatially varying coefficients, results from this method is limited to changes in modal dispersions caused by the presence of such near-wellbore stresses. Results provide changes in both the Stoneley and flexural dispersions caused by an increases in borehole overpressure, effective overburden, maximum and minimum horizontal stresses. The increase in formation shear velocity caused by a given increase in the overburden stress is the same as that caused by the horizontal stress of the same magnitude and parallel to the radial polarization. In contrast, changes in flexural velocities at high frequencies caused by an increase in the overburden stress are similar to that caused by the horizontal stress of the same magnitude and perpendicular to the radial polarization.**

Keywords: Elastic wave propagation; formation stresses; near-wellbore stresses; borehole waves; finite-difference, time-domain method; Perturbation method; Stoneley and dipole dispersions; tool effects on dipole dispersions

I. INTRODUCTION

Sedimentary rocks are highly porous and permeable. They exhibit a high degree of non-linearity in measured strain when subjected to an external stress. Elastic wave velocities in such pre-stressed materials can change by a significantly larger amount than those in non-porous crystalline materials, such as aluminum, steel, or crystalline quartz.

Elastic waves propagating in such materials can be described by equations of motion for small dynamic field superposed on a large static bias. While static strains in a material subject to externally applied load is on the order of milli-strain, dynamic strains introduced by small amplitude elastic waves are substantially smaller on the order of tens of micro-strain. Effective elastic stiffnesses in these equations of motion are functions of static displacement gradients that can be expressed in terms of static stresses and strains in the propagating medium. The presence of a fluid-filled borehole in a pre-stressed formation causes both radially and azimuthally varying stresses in the near-wellbore region. Such heterogeneity in the near-wellbore stress distributions results in spatially varying elastic stiffnesses in the equations of motion for small amplitude elastic waves. Consequently, it is necessary to use either a finite-difference, time-domain (FDTD) formulation of such equations of motion or a perturbation method for studying the influence of such static stresses on the velocity of plane waves and velocity dispersions of borehole guided modes. The lowest-order axi-symmetric Stoneley and flexural modes are easily excited in a fluid-filled borehole. Analyses of these modal dispersions are extensively used in the estimation of formation mechanical properties.

This paper describes results from the FDTD formulation of equations of motion for small amplitude waves in the presence of near-wellbore static stress distributions [1]. The FDTD method accounts for the sonic tool effects on measured dispersions in terms of a simple heavy-fluid column with calibrated parameters that replaces a complex sonic tool structure. Synthetic waveforms from the FDTD formulation are processed by a Slowness-Time-Coherence (STC) algorithm to determine the compressional velocity or slowness in the formation surrounding a fluid-filled borehole [2]. In addition, we use a modified matrix pencil algorithm that helps in isolating both non-dispersive compressional head-waves and dispersive borehole guided modes [3]. We compare results for the borehole flexural mode dispersions obtained from the FDTD formulation with those from a perturbation method.

II. Wave Propagation in a Prestressed Formation

The propagation of small amplitude waves in homogeneous and anisotropic materials is described by the linear equations of motion. However, when the material is pre-stressed, the propagation of such waves is then described by equations of motion for small dynamic fields superposed on a static bias. A static bias represents any statically deformed state of the propagating medium caused by an externally applied load above and beyond that exist in a chosen reference state. Equations of motion for small dynamic fields superposed on a static bias are derived from the rotationally invariant equations of nonlinear elasticity. The presence of a fluid-filled borehole in a tectonically stressed formation causes both radial and azimuthal heterogeneities in rock stresses. Under these circumstances, the effective elastic coefficients become position dependent and it becomes necessary to use a perturbation model or a finite-difference formulation of equations of motion for obtaining changes in velocities caused by such stress distributions. This perturbation model relates fractional changes in the modal velocities caused by given changes in the static stress distributions as a function of frequency [4, 5]. A perturbation integral equation serves as a basis for the inversion model that has been used for estimating formation stress magnitudes using measured sonic velocities in a formation subject to tectonic stresses.

II. Computational Results

The formation stress state is characterized by the magnitude and direction of three principal stresses. Figure 1 shows a schematic diagram of a vertical borehole in a formation subject to the three principal stresses. Generally, the overburden stress (SV) is reliably obtained by integrating the formation mass density from the surface to the depth of interest. Consequently, estimating the other two principal stresses (SHmax and Shmin) in the horizontal plane is the remaining task necessary to fully characterize the formation stress state.

Near-wellbore stress distributions in the borehole cross-section can be expressed in terms of far-field formation stresses [1]. Figure 2 displays radial and azimuthal variations of the radial, hoop, axial, and shear stresses surrounding a borehole of radius ‘a’. Large hoop stress $\sigma_{\theta\theta}$ (see Figure 2b) occurs at an azimuth perpendicular to the maximum horizontal stress direction. Effects of these cylindrical stresses on elastic wave propagation in a fluid-filled borehole can be studied by a finite-difference formulation of equations of motion for small dynamic fields superposed on a bias [1].

Sonic measurements in a borehole are carried out with the help of a monopole or dipole transmitter and an array of hydrophone receivers mounted on a complex tool structure. We replace this tool structure by an equivalent heavy-fluid column concentrically placed on the borehole axis that adequately accounts for the tool effects on the acquired sonic data.

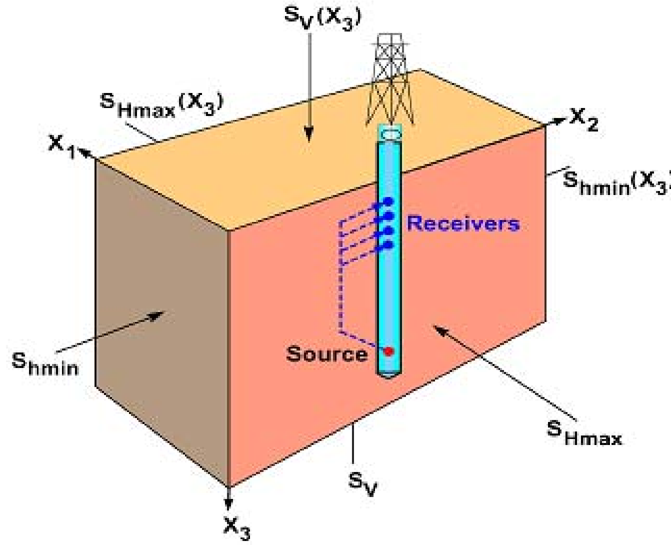

Figure 1: Schematic of a borehole in the presence of formation principal stresses with the borehole axis parallel to the overburden stress.

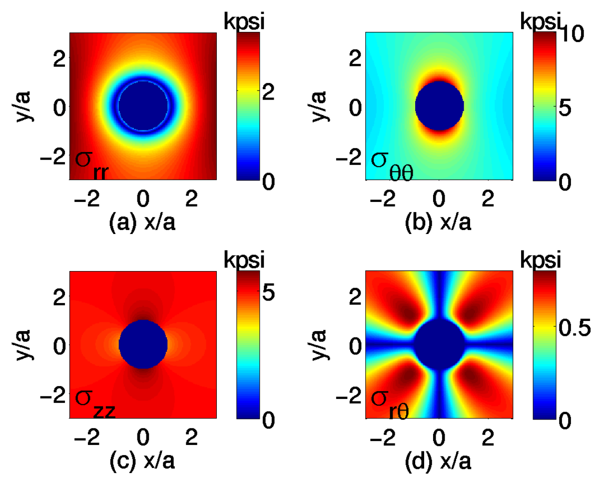

Figure 2: (a) Radial, (b) hoop, (c) axial, and (d) shear stresses in the borehole cross-sectional plane for a given wellbore pressure, overburden, maximum, and minimum horizontal stresses.

Synthetic waveforms have been obtained at an array of receivers generated by a monopole or dipole transmitters placed on the borehole axis. Table 1 contains the formation material parameters used in this study. The borehole fluid compressional velocity is 1600 m/s, and its mass density is 1050 kg/m^3. The circular borehole diameter is 20 cm.

TABLE 1. Formation material parameters

ρ_f kg/m^3	V_P m/s	V_S m/s	C_{111} TPa	C_{112} TPa	C_{123} TPa
2062	2320	1500	-21.2	-3.04	2.36

Monopole waves in formations with pre-stress

We follow the convention that the overburden stress S_{11} is parallel to the borehole axis; S_{22} and S_{33} correspond the maximum and minimum horizontal stress directions, respectively. Figure 3 shows monopole waveforms at a subarray of 10th through 17th receivers. The receiver distance is measured from the transmitter location along the borehole axis. The red and blue curves denote waveforms obtained in a formation in the absence and presence of far-field stresses (S_{11} = - 5 MPa).

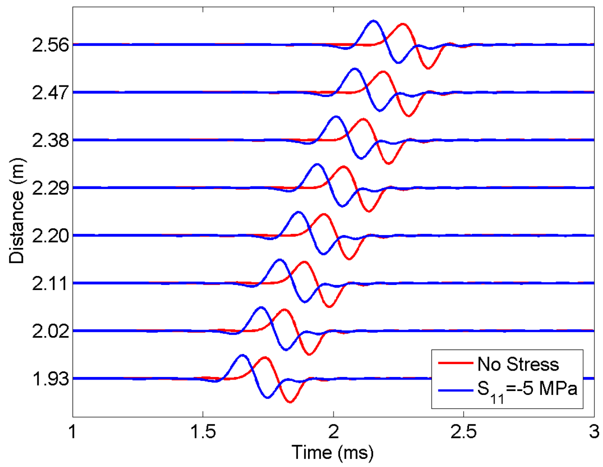

Figure 3: Monopole waveforms in the absence (red) and presence (blue) of the overburden stress.

These synthetic waveforms are then processed by a STC algorithm to estimate the compressional headwave slowness. In addition, we also process these waveforms by a modified matrix pencil algorithm that separates both the dispersive and non-dispersive arrivals in the wavetrain. While small amplitude compressional headwave signals are reliably processed by the STC algorithm, the matrix pencil algorithm fails to extract the compressional slowness when the signal amplitude is substantially smaller than the dominant Stoneley or flexural arrivals. Figure 4 displays the Stoneley dispersions obtained for the cases of (a) No formation stress; (b) Wellbore pressure P_w = - 5 MPa; (c) Overburden S_{11} = -5 MPa; (d) Horizontal stress S_{22} = -5 MPa; (e) Horizontal stress S_{33} = -5 MPa; and (f) All of these stress components $S_{11} = S_{22} = S_{33}$ = -5 MPa. Computational results indicate that the Stoneley dispersion at high frequencies is affected more by the near-wellbore stresses caused by the wellbore pressure. In contrast, the Stoneley dispersions remains essentially non-dispersive in the presence of near-wellbore stress distributions caused by the far-field formation stresses, because the sum of the hoop and radial stresses is nearly the same. We also observe that results for the case of all stress components are approximately linear superposition of results for the individual stress components.

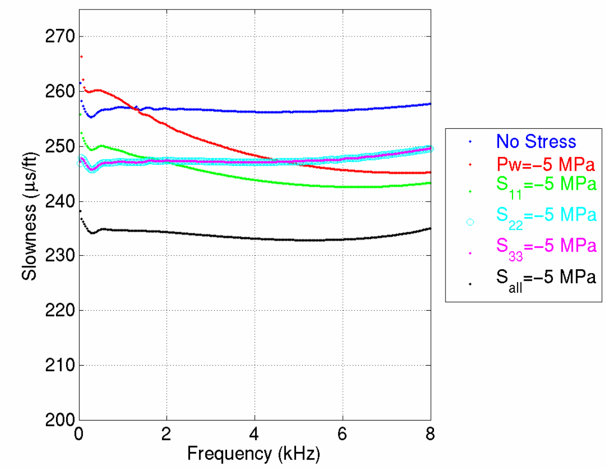

Figure 4: Monopole Stoneley dispersions in the presence of different stress components.

Dipole waves in formations with pre-stress

Figure 5 shows dipole waveforms at a subarray of 10th through 17th receivers with receiver distance measured from the transmitter location. The red and blue curves represent waveforms obtained in a formation in the absence and presence of far-field stress (S_{11} = - 5 MPa).

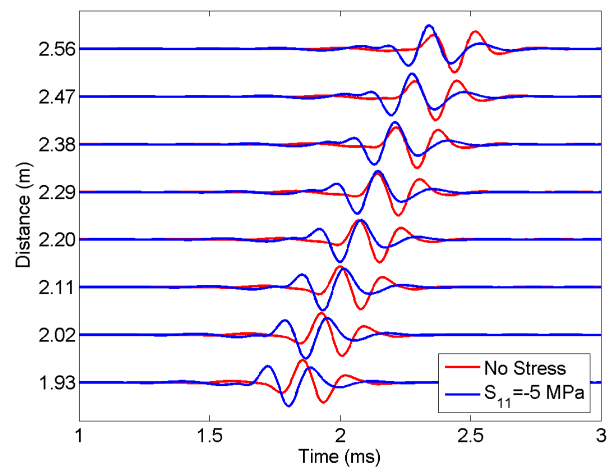

Figure 5: Dipole waveforms in the absence (red) and presence (blue) of the overburden stress.

STC processing of synthetic dipole waveforms yields compressional and shear headwave slownesses identified by semblance peaks as shown in Figure 6c (lower-left subplot). We compare in the lower-right subplot, flexural dispersions for dipole transmitter oriented parallel to the maximum horizontal stress S_{22} direction in green with that obtained for the transmitter oriented parallel to the S_{33} direction in blue. These two dipole dispersions show a characteristic crossover even in the presence of a heavy-fluid column used to compensate for the influence of a complex tool structure on the recorded dipole waveforms.

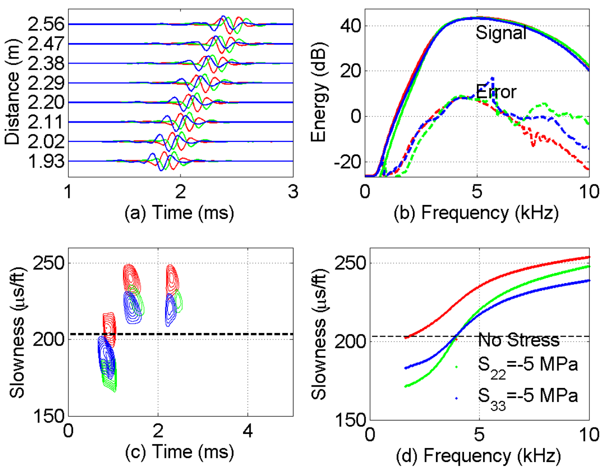

Figure 6: (a) Dipole waveforms; (b) Signal vs. fitting error; (c) Semblance peak from STC; (d) Dipole dispersions. No stress (red), S_{22}= -5 MPa (green), S_{33}= -5 MPa (blue).

Comparison of FDTD and perturbation results

We use a perturbation method for calculating changes in the Stoneley and dipole dispersions caused by the presence of different stress components in the surrounding formation. The reference state in the perturbation integral is defined by an eigensolution obtained in the presence of a heavy-fluid column that accounts for the tool effects on sonic waveforms. Figure 7 compares results for the Stoneley dispersion obtained after processing the FDTD waveforms by a modified matrix pencil algorithm with those from a perturbation method. Good agreement is obtained from these two techniques for calculating stress-induced effects on Stoneley dispersions.

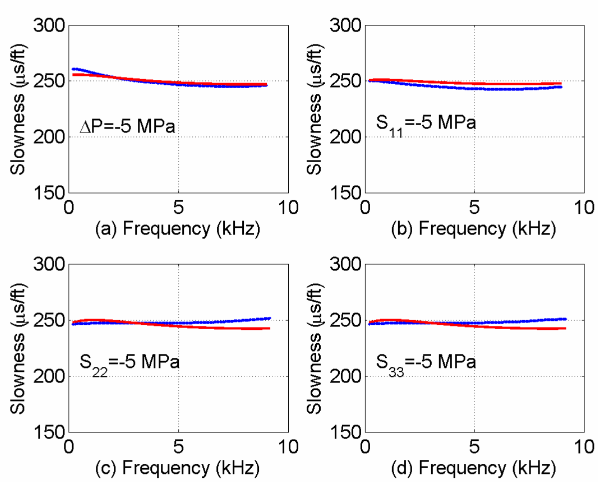

Figure 7: Comparison of the Stoneley dispersions obtained from FDTD waveforms (blue) and a perturbation method (red). (a) P_w = - 5 MPa; (b) S_{11} = -5 MPa; (c) S_{22} = -5 MPa; (d) S_{33} = -5 MPa.

Similarly, Figure 8 shows comparison of dipole dispersions obtained from the FDTD waveforms and those from a perturbation method that includes the presence of a heavy-fluid column.

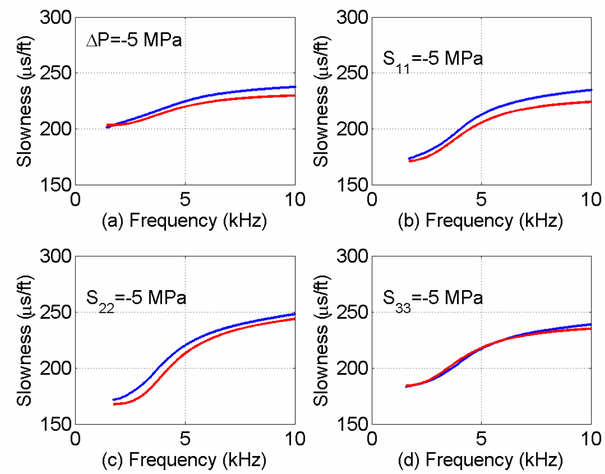

Figure 8: Comparison of the dipole dispersions obtained from FDTD waveforms (blue) and a perturbation method (red). (a) P_w = - 5 MPa; (b) S_{11} = -5 MPa; (c) S_{22} = -5 MPa; (d) S_{33} = -5 MPa.

III. CONCLUSIONS

Computational results have been obtained for changes in the lowest-order axi-symmetric and flexural mode dispersions caused by the presence of formation tectonic stresses. Sonic tool effects are accounted for by introducing an equivalent heavy-fluid column with calibrated parameters. Good agreement is observed for borehole dispersions in the presence of formation stresses obtained from the processing of synthetic waveforms using a 3D-cylindrical finite-difference formulation as well as a perturbation model.

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