Application of the Hybrid Spectral Integral Method with Spectral Element Method

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Exact radiation boundary conditions are of great interest to the numerical solution of Maxwell's equations for an unbounded domain. Previously, the boundary element method has been used as an exact radiation boundary condition in the finite element method. This technique is useful and provides an accurate solution to inhomogeneous objects if the sampling density is high enough. To further improve the accuracy and efficiency of this technique, we have developed a high-order version of such a hybrid technique, i.e., an exact radiation boundary condition for the spectral element method (SEM) based on the spectral integral method (SIM) for three-dimensional electromagnetic problems. This work will demonstrate the high-accuracy and efficiency of the hybrid SIM/SEM technique for 3D scattering problems.

First, we have developed a spectral integral method for 3D surface integral equations in electromagnetics. Previously, SIM techniques have been investigated by our group and several researchers and the spectral accuracy has been demonstrated, thus showing a great potential to large-scale applications. In this work, we further extend the SIM to the 3D vector case. We show that the spectral integral method works efficiently for homogeneous objects.

Furthermore, we have recently developed a spectral element method (SEM) to achieve spectral accuracy with the orders of the basis functions. Previously, the finite element method has been widely used to solve inhomogeneous and non-smooth problems, but its accuracy order remains low because of its low-order basis functions. The SEM can be considered as a special class of the general finite element method, with the choice of nodal points and quadrature integration points based on Gauss-Lobatto-Legendre (GLL) polynomials. Thus, the SEM has versatility in modeling inhomogeneous materials and can give high-order accuracy. Moreover, the numerical integration of in the SEM utilizes the GLL quadrature points, thus removing the Runge errors usually associated with high-order methods.

In the proposed hybrid SIM/SEM technique, the spectral integral method is applied on the exterior boundary of a finite domain, while the spectral element method is applied to the interior domain containing contains inhomogeneous materials. This allows complex objects to be modeled efficiently. Furthermore, the hybrid SIM/SEM uses the fast Fourier transform (FFT) algorithm to accelerate the matrix-vector multiplication, thus greatly reducing the computational cost compared to the hybridization of the boundary element method with the finite element method. Numerical examples with highly inhomogeneous media will be shown to demonstrate the efficacy of this hybrid method.

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