CMSC 635

Sampling and Antialiasing

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Abstract Vector Spaces

- Addition
  - C = A + B = B + A
  - (A + B) + C = A + (B + C)
  - given A, B, A + X = B for only one X
- Scalar multiply
  - C = a A
  - a (A + B) = a A + a B
  - (a+b) A = a A + b A

Abstract Vector Spaces

- Inner or Dot Product
  - $b = a\,(A \cdot B) = a\,A \cdot B = A \cdot a\,B$
  - $A \cdot A \geq 0$; $A \cdot A = 0$ iff $A = 0$
  - $A \cdot B = (B \cdot A)^*$

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Vectors and Discrete Functions

Vector	Discrete Function
V = (1, 2, 4)	V[I] = {1, 2, 4}
a V + b U	a V[I] + b U[I]
V • U	$\sum$ (V[I] U^*[I])

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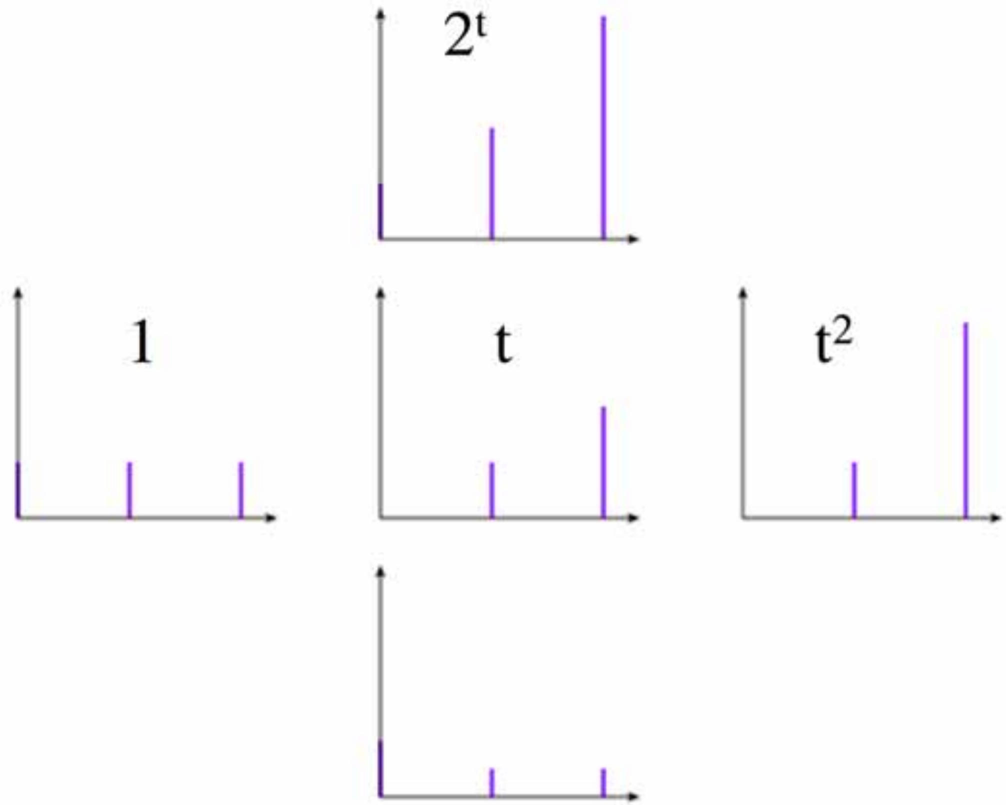
Vectors and Discrete Functions

- 2^t in terms of t^0, t^1, t^2 = [1,.5,.5]



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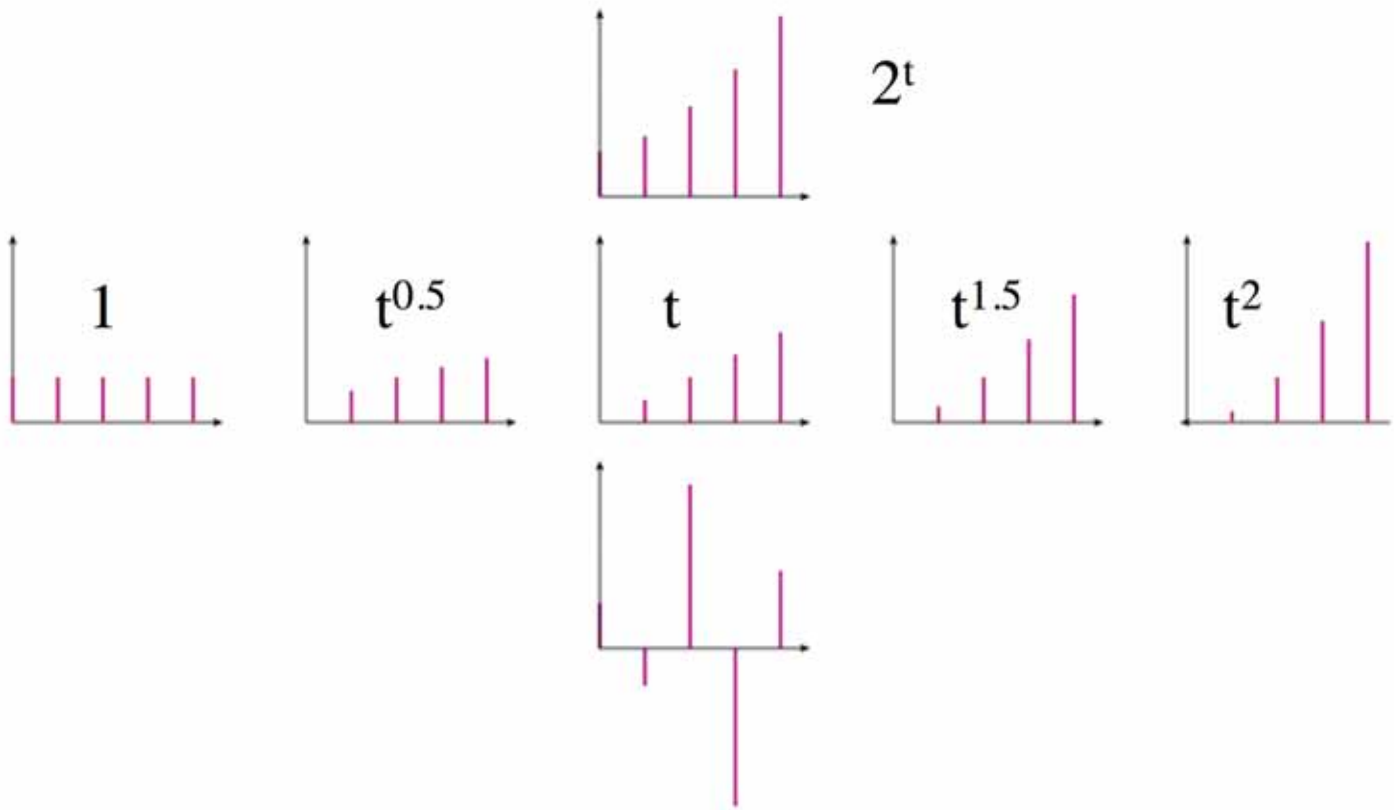
Vectors and Discrete Functions

- 2^t in terms of t^0, $t^{0.5}$, t^1, $t^{1.5}$, t^2



Vectors and Functions

Vector	Discrete	Continuous
V	$V[I]$	$V(x)$
$a V + b U$	$a V[I] + b U[I]$	$a V(x) + b U(x)$
$V \bullet U$	$\sum V[I] U^*[I]$	$\int V(x) U^*(x) dx$

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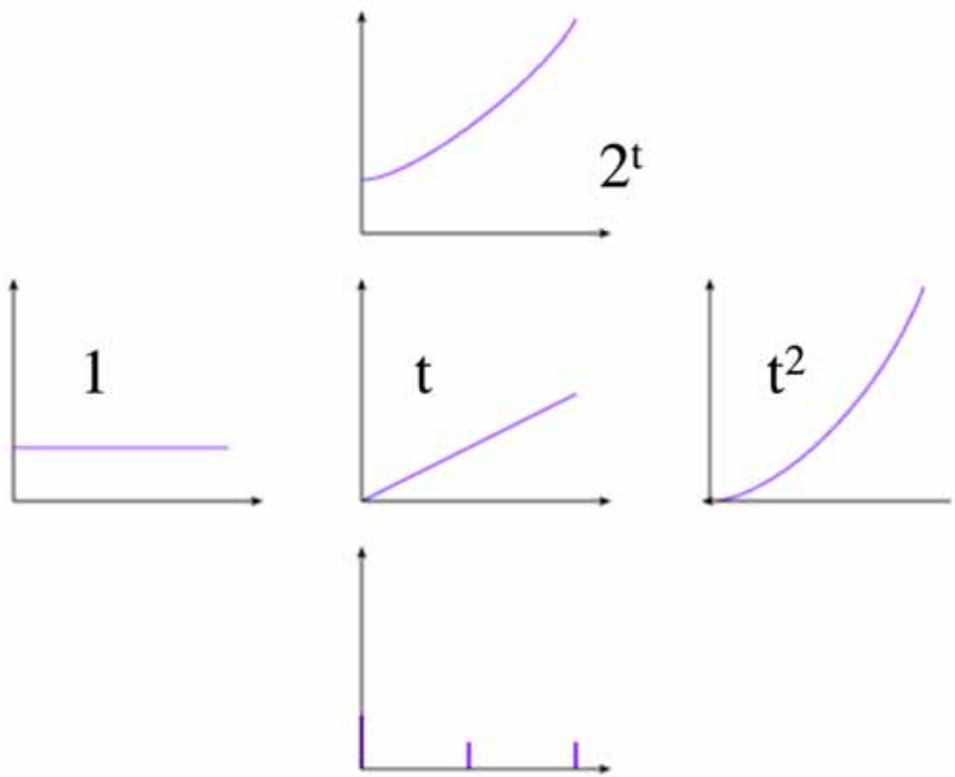
Vectors and Functions

- 2^t projected onto 1, t, t^2



Function Bases

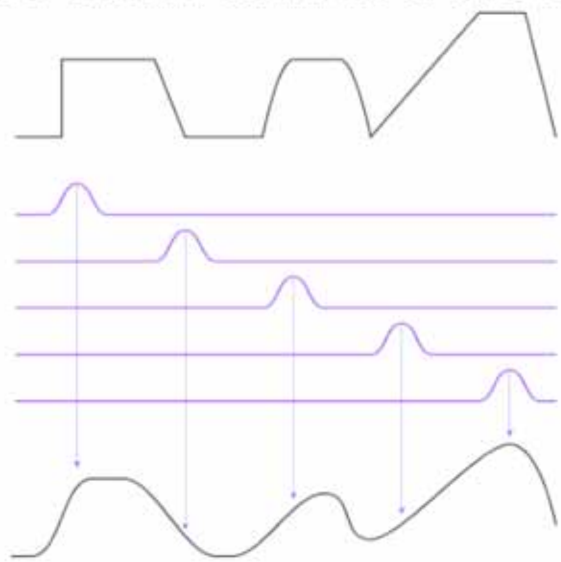
- Time: $\delta(t)$
- Polynomial / Power Series: t^n
- Discrete Fourier: $e^{i \pi t K/N} / \sqrt{2N}$
  - K, N integers
  - $t, K \in [-N, N]$
  - (where $e^{i\theta} = \cos\theta + i \sin\theta$)
- Continuous Fourier: $e^{i \omega t} / \sqrt{2\pi}$

Fourier Transforms

	Discrete Time	Continuous Time
Discrete Frequency	Discrete Fourier Transform	Fourier Series
Continuous Frequency	Discrete-time Fourier Transform	Fourier Transform

Convolution

- $f(t)\, g(t) \Leftrightarrow F(\omega) * G(\omega)$
- $g(t) * f(t) \Leftrightarrow F(\omega)\, G(\omega)$
- Where $f(t) * g(t) = \int f(s)\, g(t-s)\, ds$
  - Dot product with shifted kernel



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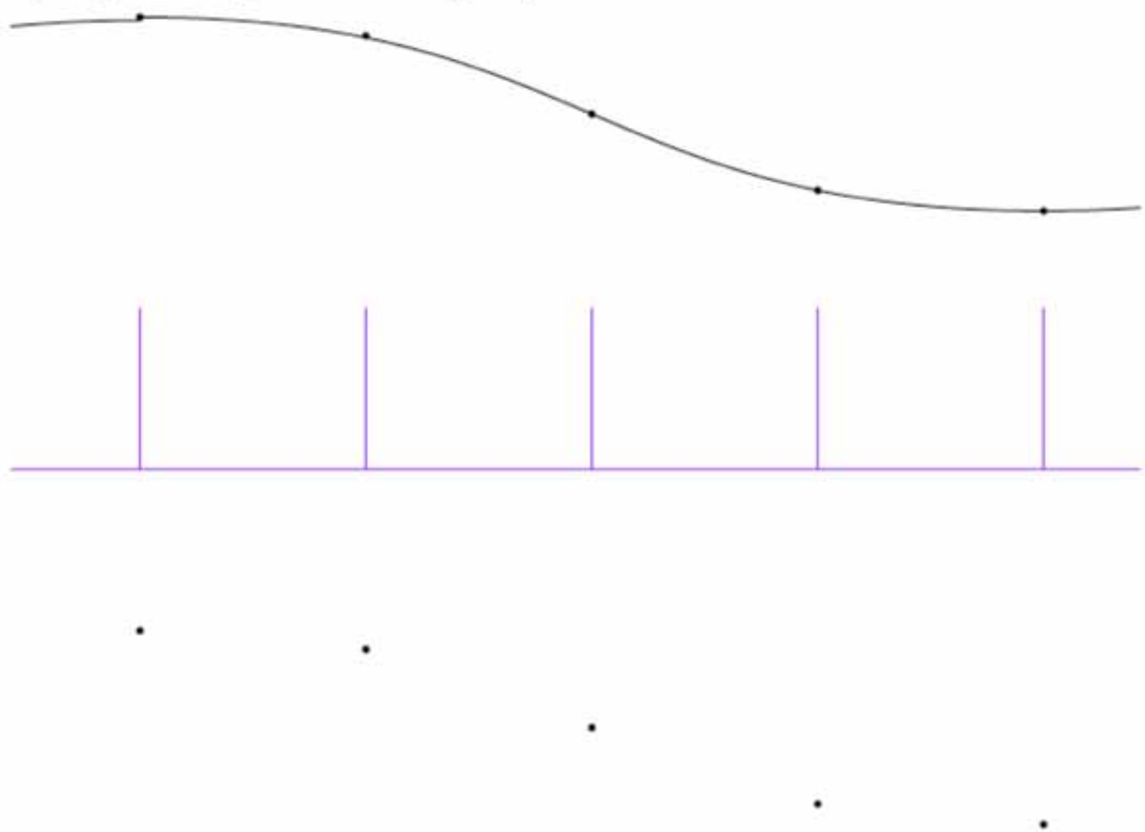
Filtering

- Filter in frequency domain
  - FT signal to frequency domain
  - Multiply signal & filter
  - FT signal back to time domain
- Filter in time domain
  - FT filter to time domain
  - Convolve signal & filter

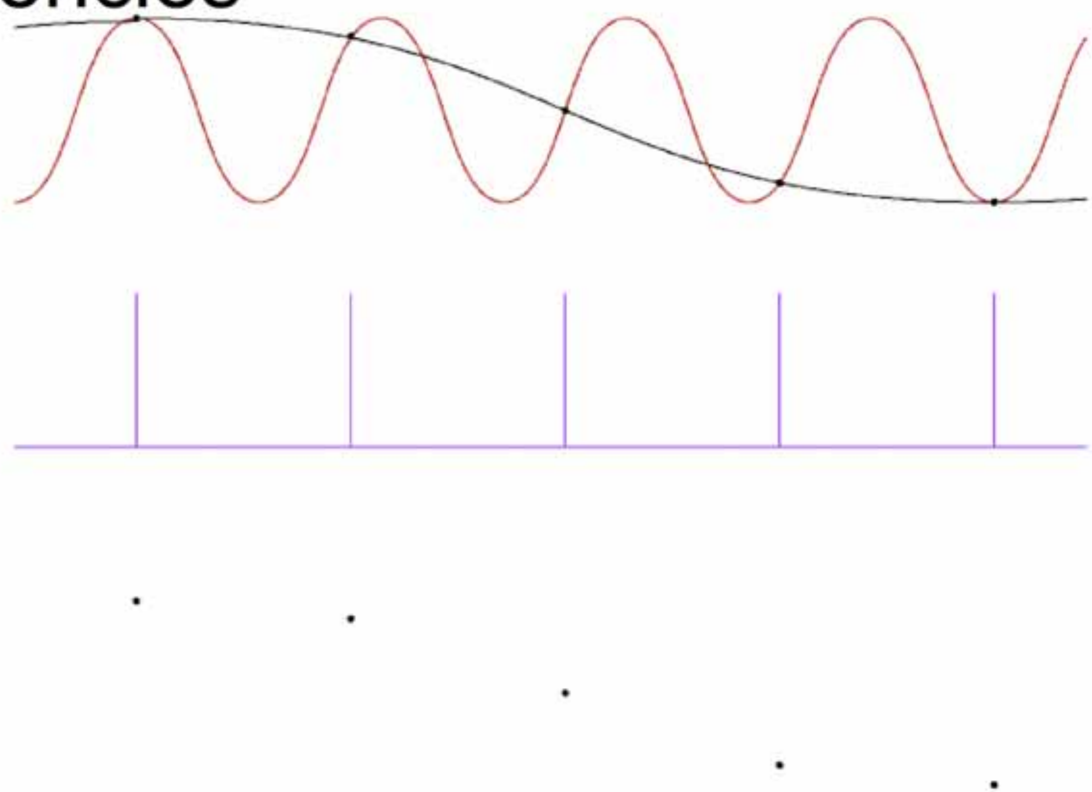
Sampling

- Multiply signal by pulse train

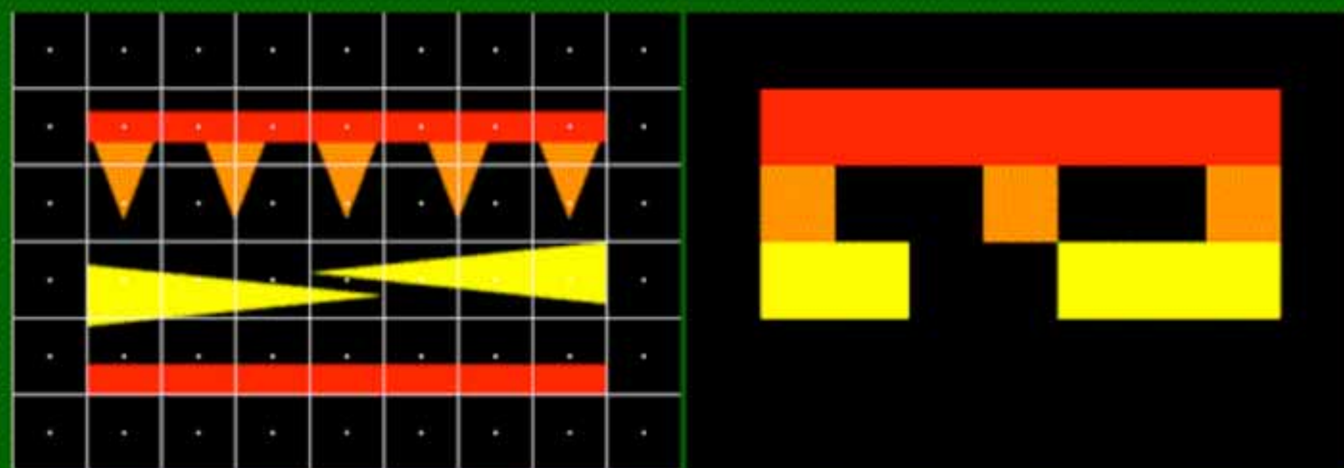


Aliasing

- High frequencies alias as low frequencies



Aliasing in images



Original *Rendered*

Loss of detail

Antialiasing

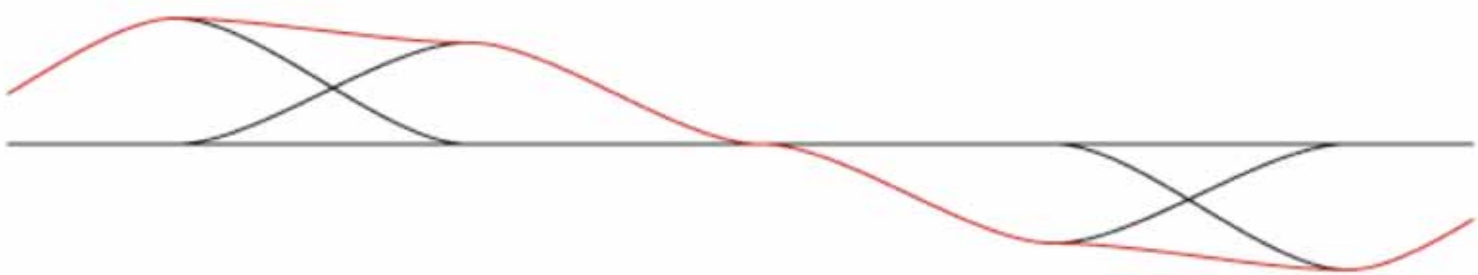
- Blur away frequencies that would alias
- Blur preferable to aliasing
- Filter kernel size
  - IIR = infinite impulse response
  - FIR = finite impulse response
    - Windowed filters

Ideal

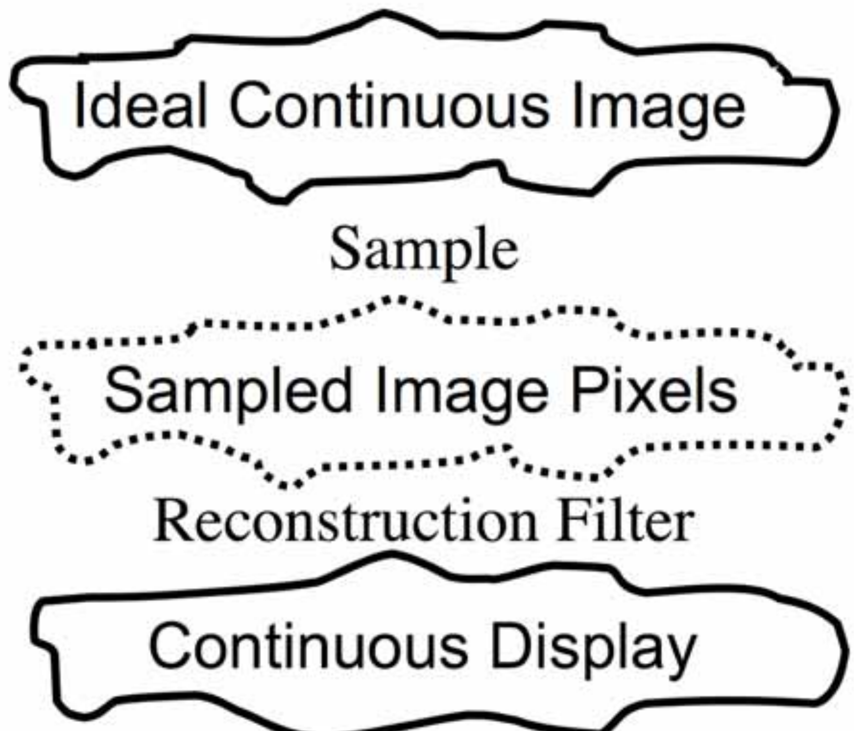
- Low pass filter eliminates all high freq
  - box in frequency domain
  - sinc in spatial domain (sin x / x)
  - Possible negative results
  - Infinite kernel
- Exact reconstruction to Nyquist limit
  - Sample frequency ≥ 2x highest frequency
  - Exact only if reconstructing with ideal low-pass filter (=sync)

Reconstruction

- Convolve samples & reconstruction filter
- Sum weighted kernel functions



Filtering & Reconstruction



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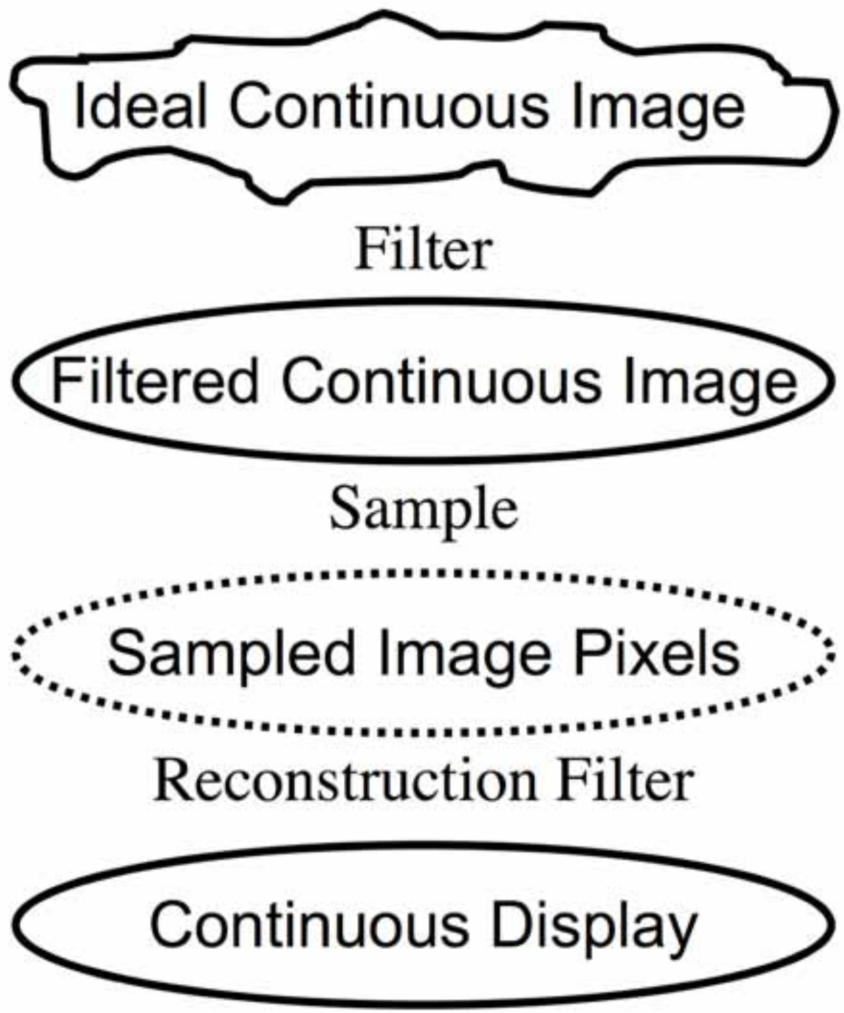
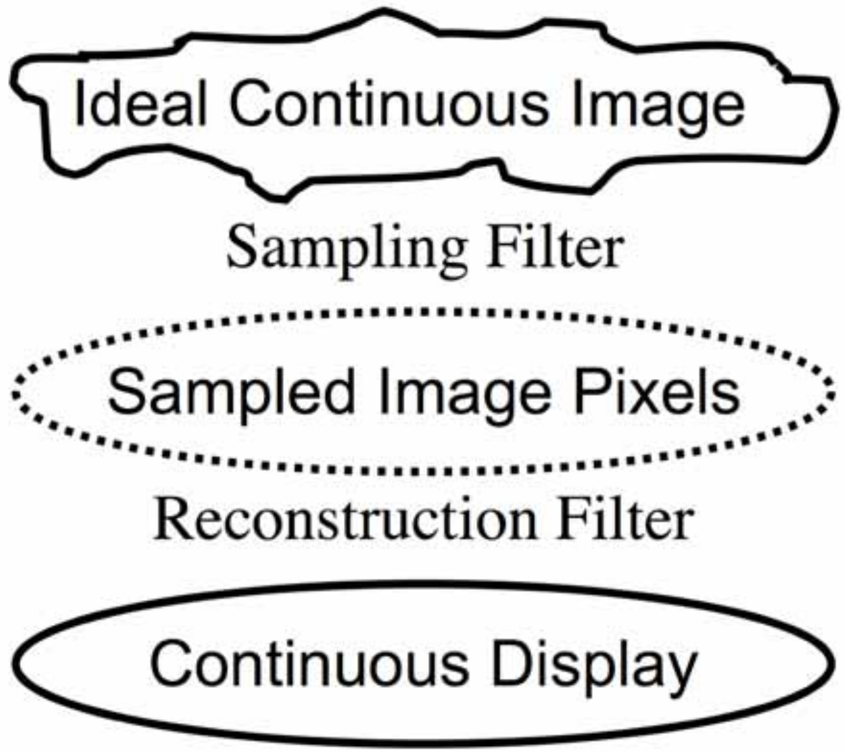
Filtering, Sampling, Reconstruction



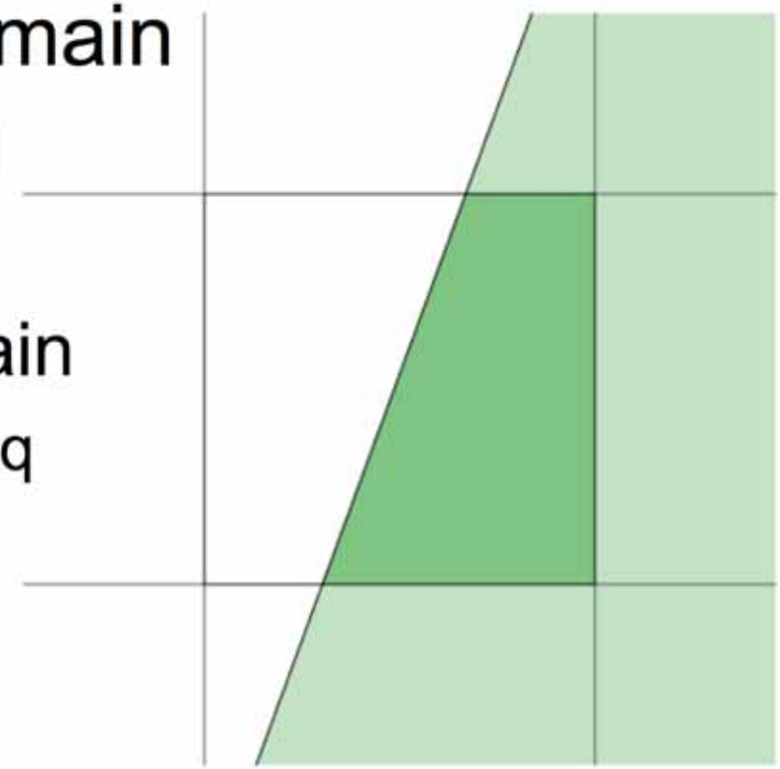
Combine Filter & Sample

- Can combine filter and sample
  - Evaluate convolution at samples



Analytic Area Sampling

- Compute “area” of pixel covered
- Box in spatial domain
  - Nice finite kernel
    - easy to compute
  - sinc in freq domain
    - Plenty of high freq
    - still aliases



Analytic higher order filtering

- Fold better filter into rasterization
  - Can make rasterization much harder
  - Usually just done for lines
    - Draw with filter kernel "paintbrush"
- Only practical for finite filters

Supersampling

- Numeric integration of filter
- Grid with equal weight = box filter
- Other filters:
  - Grid with unequal weights
  - Priority sampling
- Push up Nyquist frequency
  - Edges: ∞ frequency, still alias

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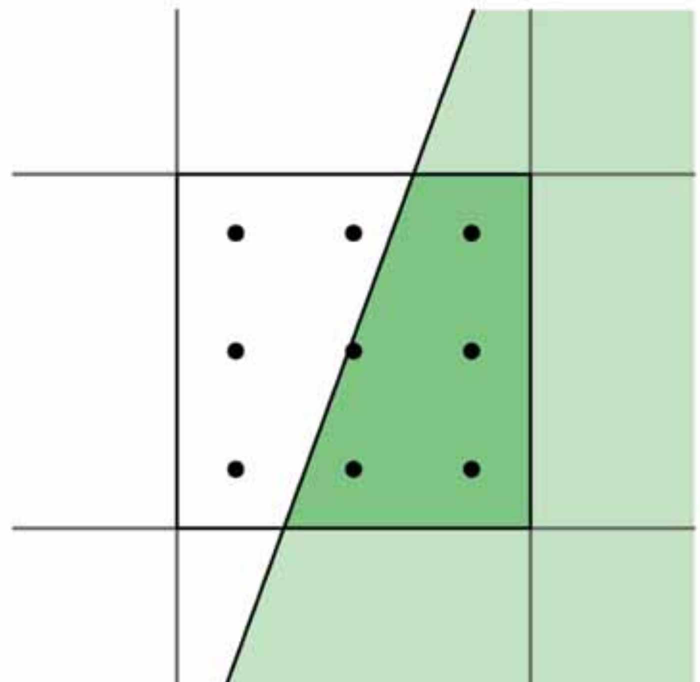

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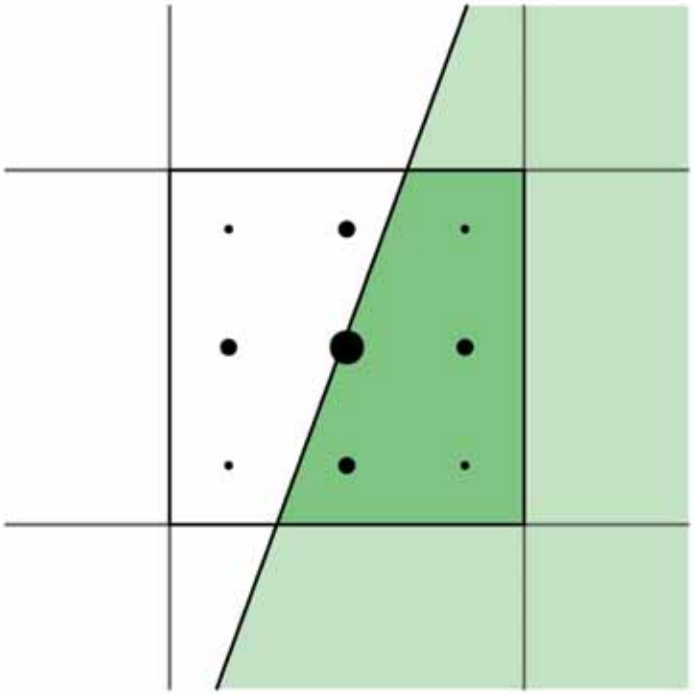

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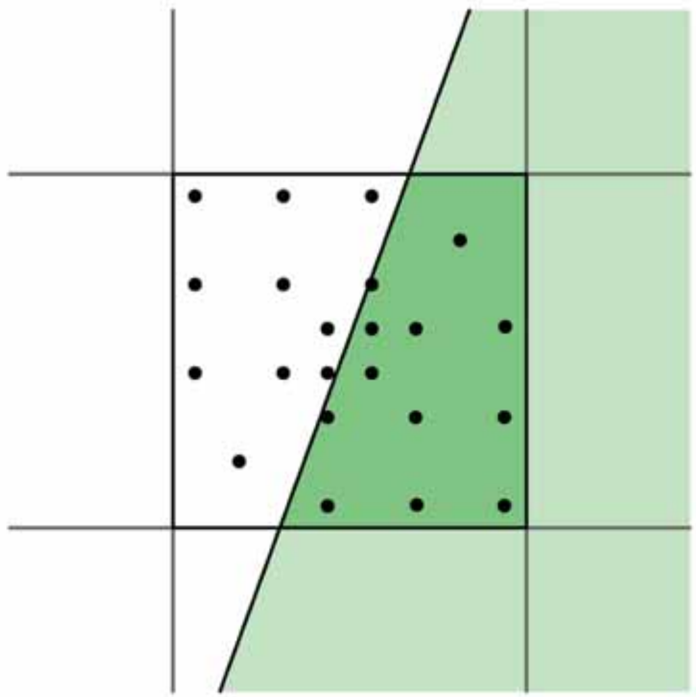

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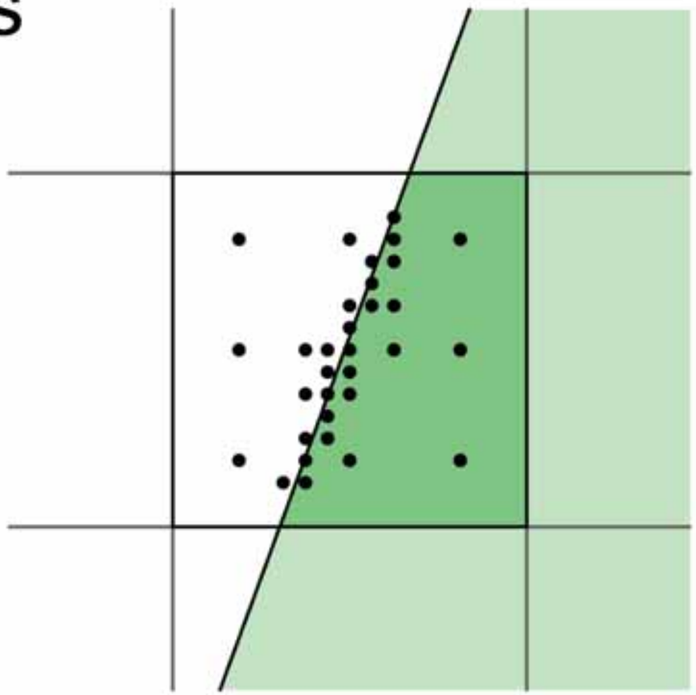
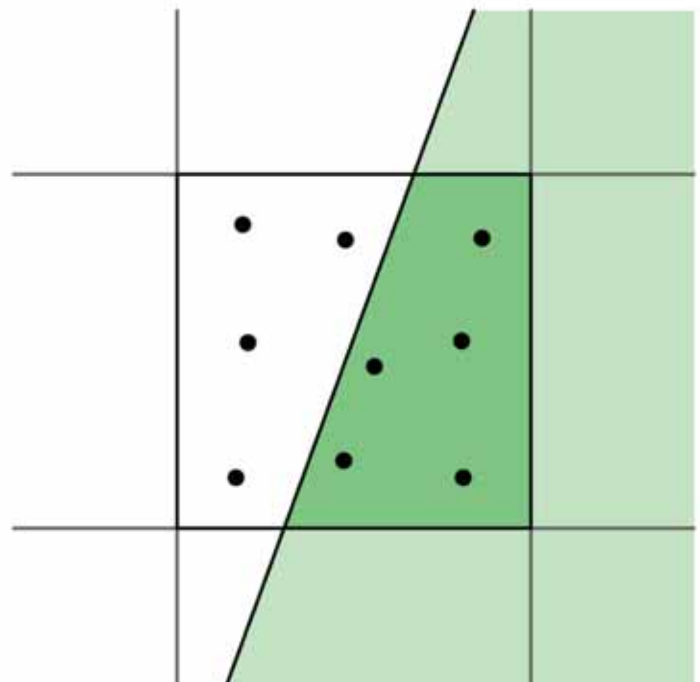
Adaptive sampling

- Vary numerical integration step
- More samples in high contrast areas
- Easy with ray tracing, harder for others
- Possible bias



Stochastic sampling

- Monte-Carlo integration of filter
- Sample distribution
  - Poisson disk
  - Jittered grid
- Aliasing ⇔ Noise



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Resampling

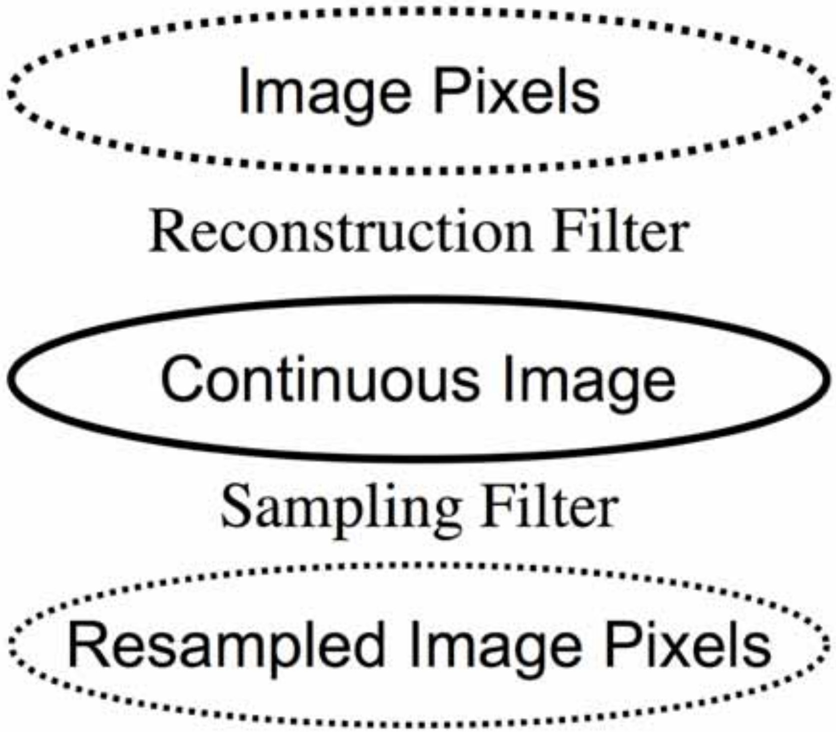
Image Pixels
Reconstruction Filter
Continuous Image
Sampling Filter
Resampled Image Pixels

Resampling