CMSC 635

Volume Rendering

Volume data

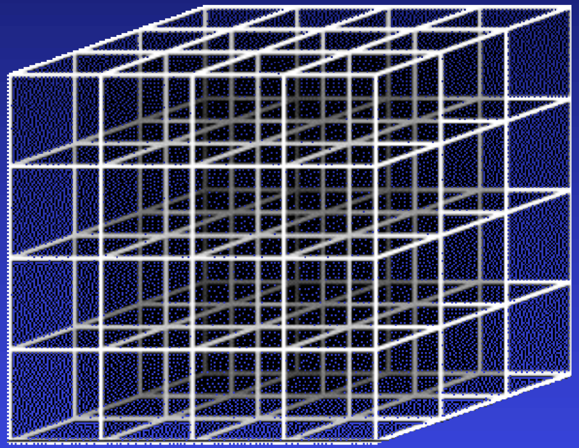
- 3D Scalar Field: F(x,y,z) = ?
  - Implicit functions
  - Voxel grid
- Scalar data
  - Density
  - Temperature
  - Wind speed
  - …

Implicit functions

- Blobs [Blinn 82]
  - $\exp(-ar^2)$
- Metaballs [Nishimura 83]
  - $0 \le r \le R_i/3 : 1 - 3(r / R_i)^2$
  - $R_i/3 \le r \le R_i : 3\,(1 - r / R_i)^2 / 2$
  - $r < R_i : 0$
- Soft Objects [Wyvill 86]
  - Polynomial approximation for exp()

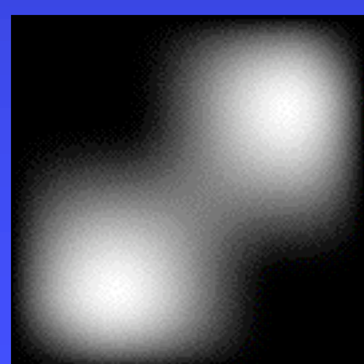
Voxels

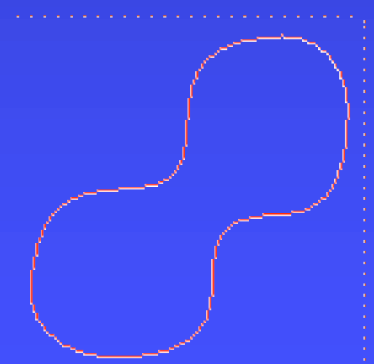
- Sampled volume
  - Usually in a grid
- Measured
  - MRI, CT scan, …
- Computed
  - Sample geometric model
  - Finite element simulation
  - …



Isosurface rendering

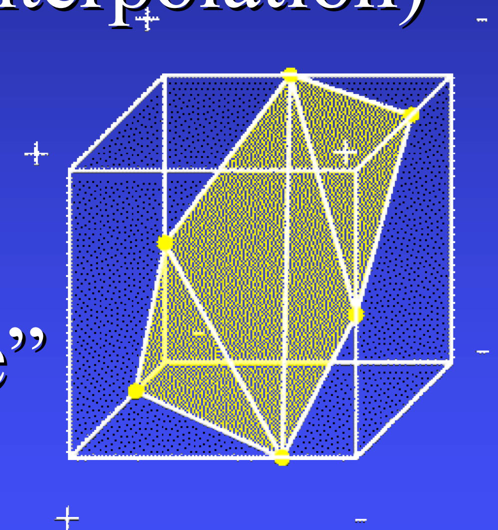
- $F(x,y,z) - c = 0$ (for some given c)
- Isosurface normal: ∇F
- Implicit: Point repulsion [Witkin 92]
- Voxel: Marching cubes [Lorensen 87]

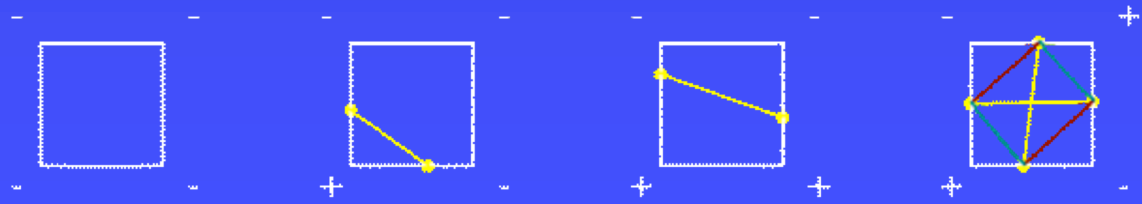




Marching cubes

- Estimate intersection point on each edge
  - Same criteria (e.g. linear interpolation)
  - Polygons will match
- Use template for polygons
  - 2^8 possibilities, 15 “unique”
  - Store templates in table





Marching tetrahedra

- Decompose volume into tetrahedra
- Avoids ambiguous “opposite corner” cases
- $2^4 = 16$ cases, 3 unique
  - 0 or 4 points inside (0 triangles)
  - 1 or 3 points inside (1 triangle)
  - 2 points inside (2 triangles)

Dividing cubes

- Find voxels that cross isosurface
- Subdivide to pixel-sized sub-voxels
- Find sub-voxels that cross isosurface
- Plot as shaded points / kernel footprints

Direct volume rendering

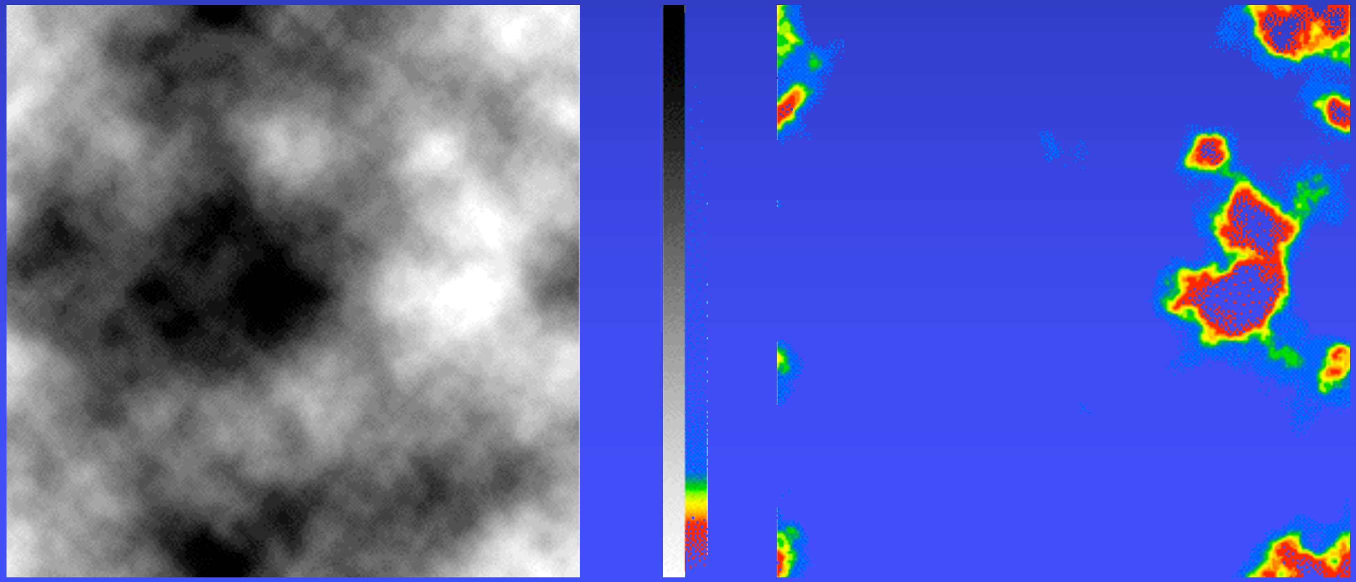
- Model as transparent material
  - Color and extinction $C(p)$, $\square(p)$
  - Attenuation along ray p: $p = r(t)$
    - $\square(t) = e^{-\int \square(t')\, dt'}$
  - Attenuated color at $r(t)$
    - $C(t)\ \square(t)$
  - Accumulate attenuated colors along ray
    - $I = \int C(t)\ \square(t)\, dt$

Simplify volume integral

- Numeric integration, step size d
  - $e^{-\int \square dt} \approx e^{-\sum \square d} = \prod e^{-\square d} = \prod (1-\square_i)$
  - Color of ray segment i $\approx \square_i C_i$
  - $I \approx \sum \square_i C_i \prod(1-\square_j)$
- Back to front composite
  - $C'_i = \square_i C_i + (1-\square_i) C'_{i+1}$

Transfer functions

- Map scalar to color and/or opacity



Appearance

- Additive / pseudo-XRay
- Volume lighting: $N \bullet L$, $(N \bullet H)^e$
  - $N \bullet V = \nabla F \bullet V$ = Directional derivative
  - $F_v \approx \big(F(P+kV) - F(P-kV)\big) / 2k$

Rendering methods

- Ray casting
- Splatting
- Texture accumulation
- Shear-warp

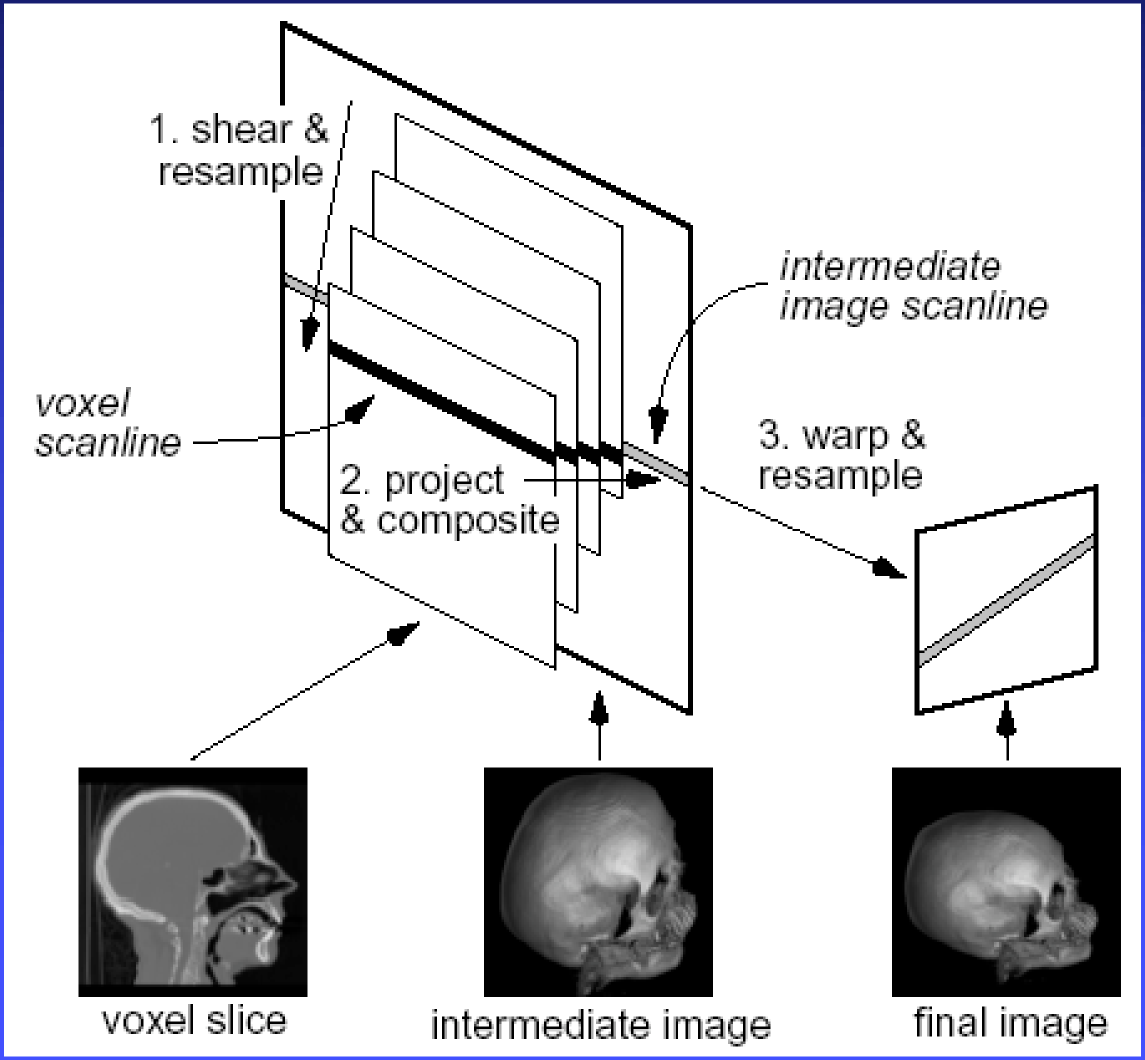
Ray casting

- Straightforward numerical integration
- Uniform steps along ray
- Resample volume to sample points
  - Before classification and/or shading
  - After classification and/or shading

Splatting [Westover 90]

- Resample directly onto screen
- Each voxel contributes kernel footprint
- Accumulate back-to-front

Shear-warp [Lacroute 94]

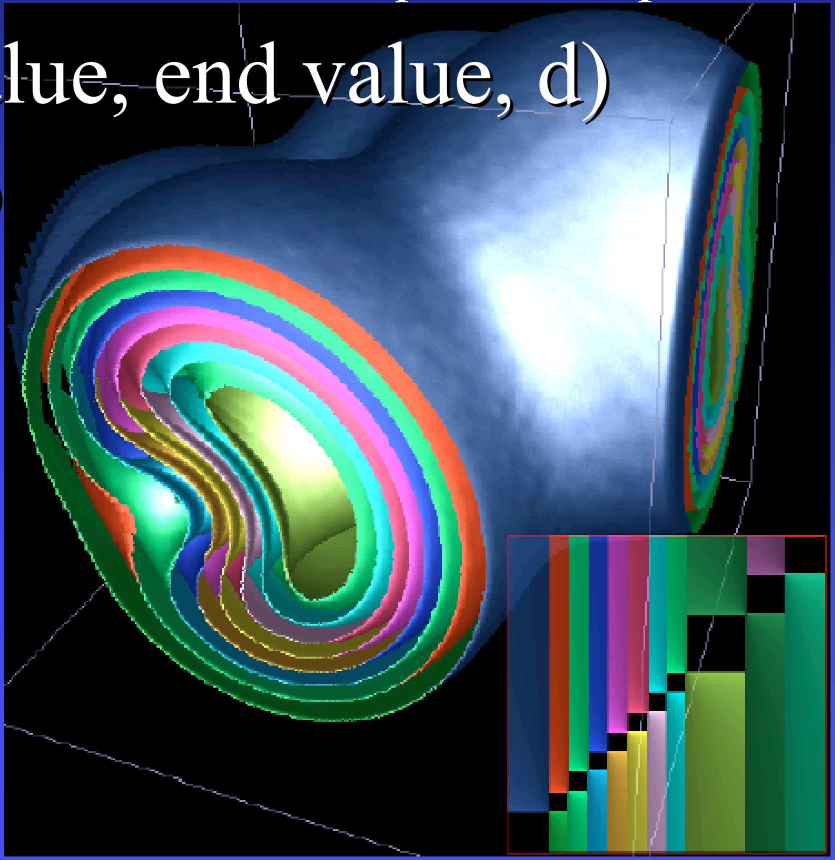


Texture accumulation

- Let texturing hardware resample
- Accumulate back-to-front
- 3D textures
  - Render slices parallel to image plane
  - Shift accesses for □F • L, □F • H
- 2D texture slices
  - Slice sets perpendicular to each axis
  - Choose set most parallel to image plane

Pre-integrated texture [Engel 01]

- Improve approximation for C_i and $\square_i$
  - Lookup(start value, end value, d)
- Dependent lookup
  - 3D texture
  - 2D texture
    - linear in d
    - constant d



Pre-integrated texture

- a: shading before resampling
- b: shading after resampling
- c: b with interpolated slices
- d: pre-integrated, same slice set as b