# CMSC 635

## Sampling and Antialiasing

# Abstract Vector Spaces

- Addition
  - $C = A + B = B + A$
  - $(A + B) + C = A + (B + C)$
  - given A, B, $A + X = B$ for only one X
- Scalar multiply
  - $C = a A$
  - $a (A + B) = a A + a B$; $(a+b) A = a A + b A$

# Abstract Vector Spaces

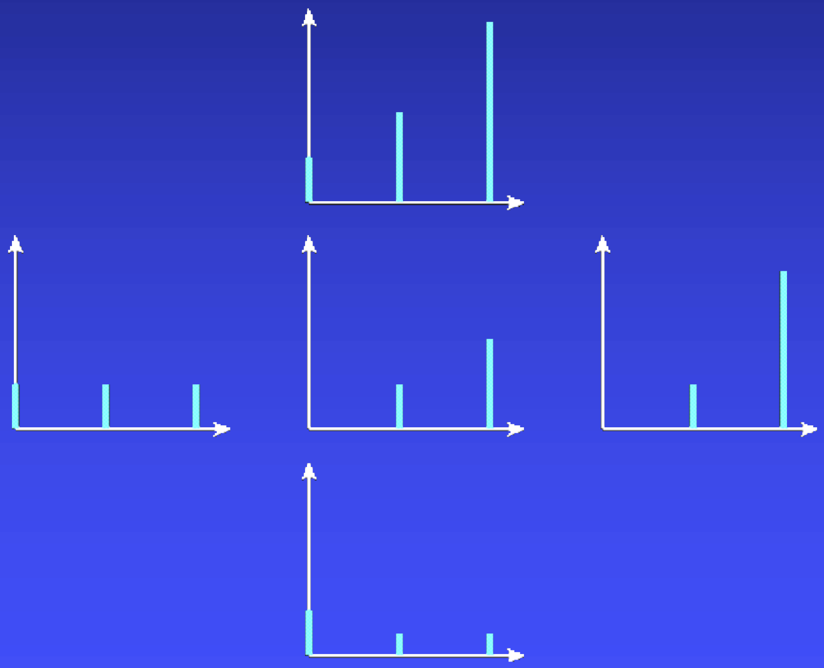
- Inner or Dot Product
  - $b = a\,(A \cdot B) = a\,A \cdot B = A \cdot a\,B$
  - $A \cdot A \geq 0$; $A \cdot A = 0$ iff $A = 0$
  - $A \cdot B = (B \cdot A)^*$

# Vectors and Discrete Functions

| Vector | Discrete Function |
|---|---|
| V = (1, 2, 4) | V[I] = {1, 2, 4} |
| a V + b U | a V[I] + b U[I] |
| V • U | $\sum$ (V[I] $U^*$[I]) |

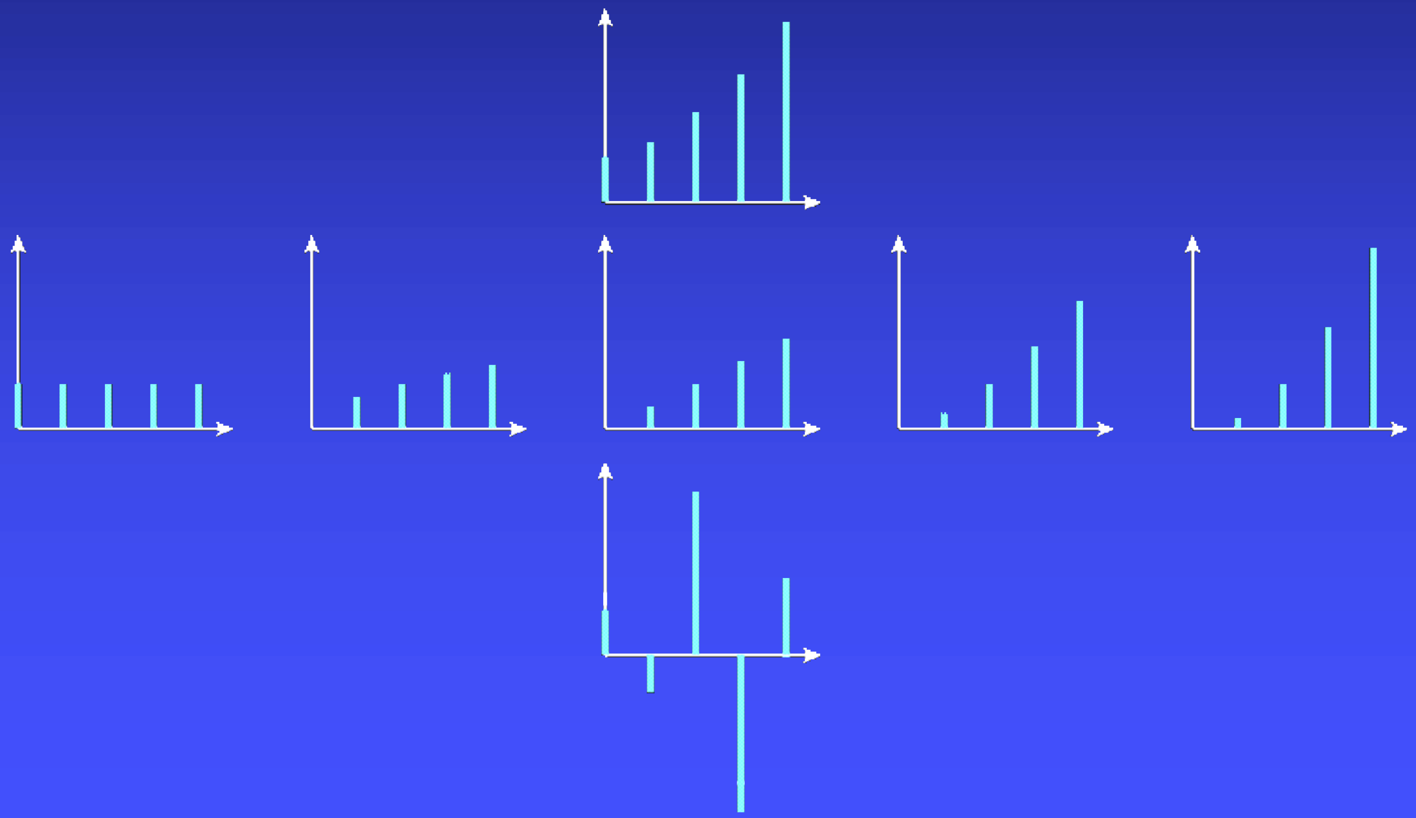
# Vectors and Discrete Functions

- $2^I$ in terms of $1, I, I^2 = [1,.5,.5]$



# Vectors and Discrete Functions

- $2^I$ in terms of $1, I^{.5}, I^2, I^{1.5}, I^4$

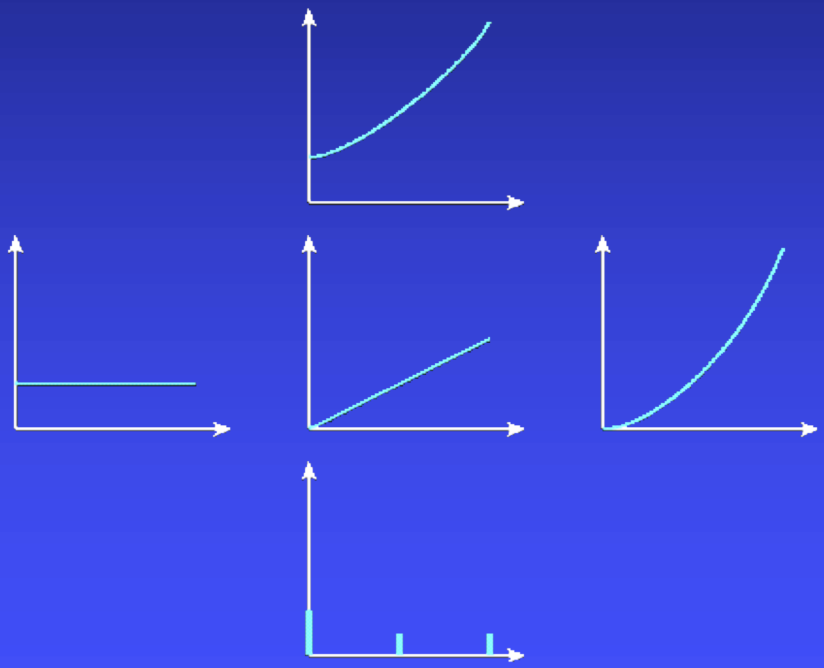


# Vectors and Functions

| Vector | Discrete | Continuous |
|---|---|---|
| $V$ | $V[I]$ | $V(x)$ |
| $a V + b U$ | $a V[I] + b U[I]$ | $a V(x) + b U(x)$ |
| $V \bullet U$ | $\sum V[I] U^*[I]$ | $\int V(x) U^*(x) dx$ |

# Vectors and Functions

- $2^x$ projected onto 1, x, $x^2$



# Function Bases

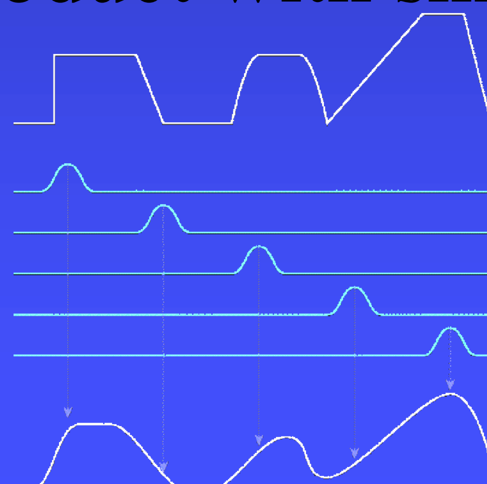
- Time: □(t)
- Polynomial / Power Series: $t^n$
- Discrete Fourier: $e^{i \pi t K/N} / \sqrt{2N}$
  - K, N integers
  - t, K □ [-N, N]
  - (where $e^{i□} = \cos □ + i \sin □$)
- Continuous Fourier: $e^{i □ t} / \sqrt{2\pi}$

# Fourier Transforms

| | Discrete Time | Continuous Time |
|---|---|---|
| Discrete Frequency | Discrete Fourier Transform | Fourier Series |
| Continuous Frequency | Discrete-time Fourier Transform | Fourier Transform |

# Convolution

- f(t) g(t) □ F(□) * G(□)
- g(t) * f(t) □ F(□) G(□)
- Where f(t) * g(t) = ∫ f(s) g(t-s) ds
  - Dot product with shifted kernel



# Filtering

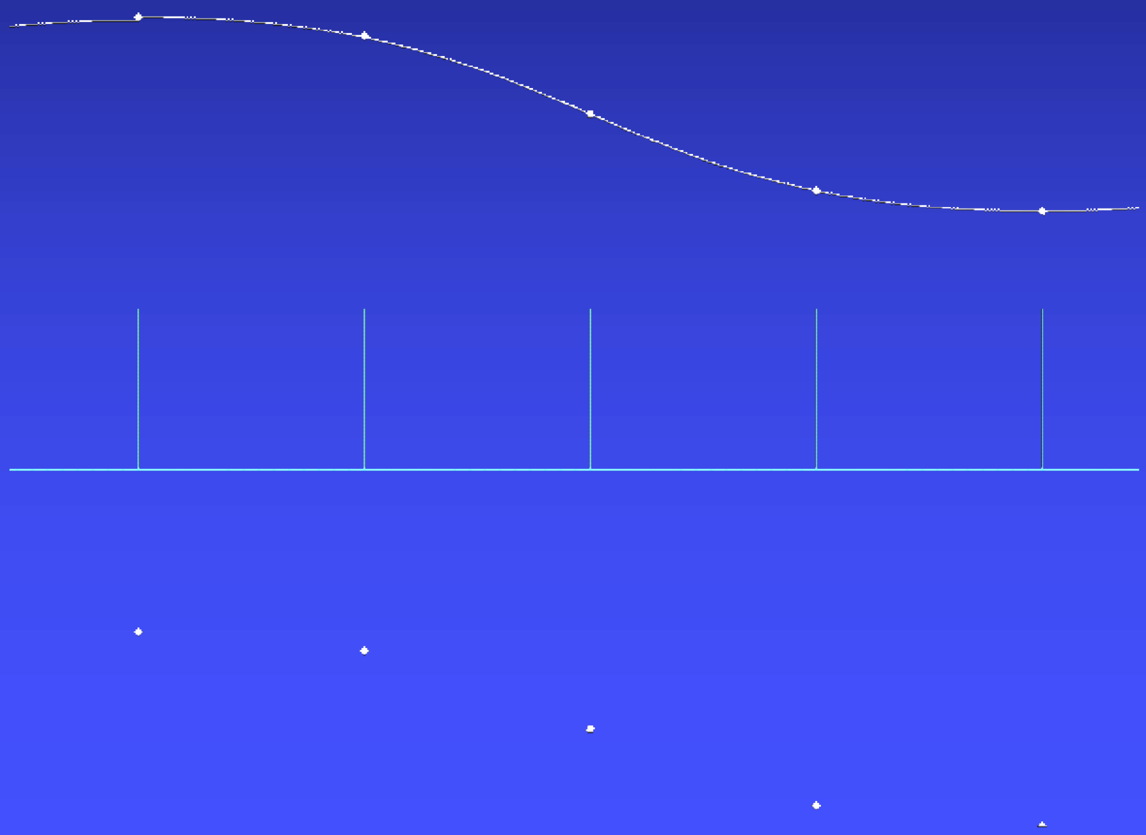
- Filter in frequency domain
  - FT signal to frequency domain
  - Multiply signal & filter
  - FT signal back to time domain
- Filter in time domain
  - FT filter to time domain
  - Convolve signal & filter

# Ideal

- Low pass filter eliminates all high freq
  - box in frequency domain
  - sinc in spatial domain (sin x / x)
  - Possible negative results
  - Infinite kernel
- Exact reconstruction to Nyquist limit
  - Sample frequency ≥ 2x highest frequency
  - Exact only if reconstructing with sync

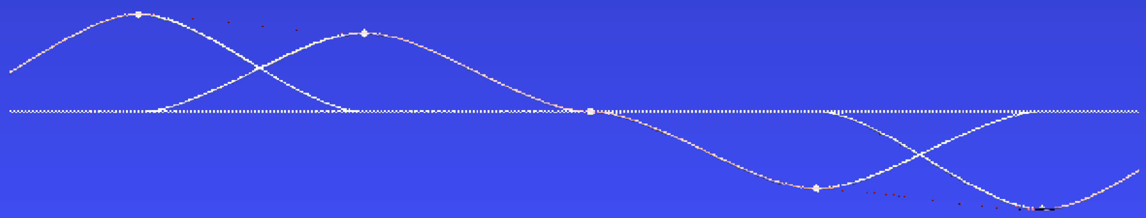
# Sampling

- Multiply signal by pulse train



# Reconstruction

- Convolve samples & reconstruction filter
- Sum weighted kernel functions



# Filtering, Sampling, Reconstruction

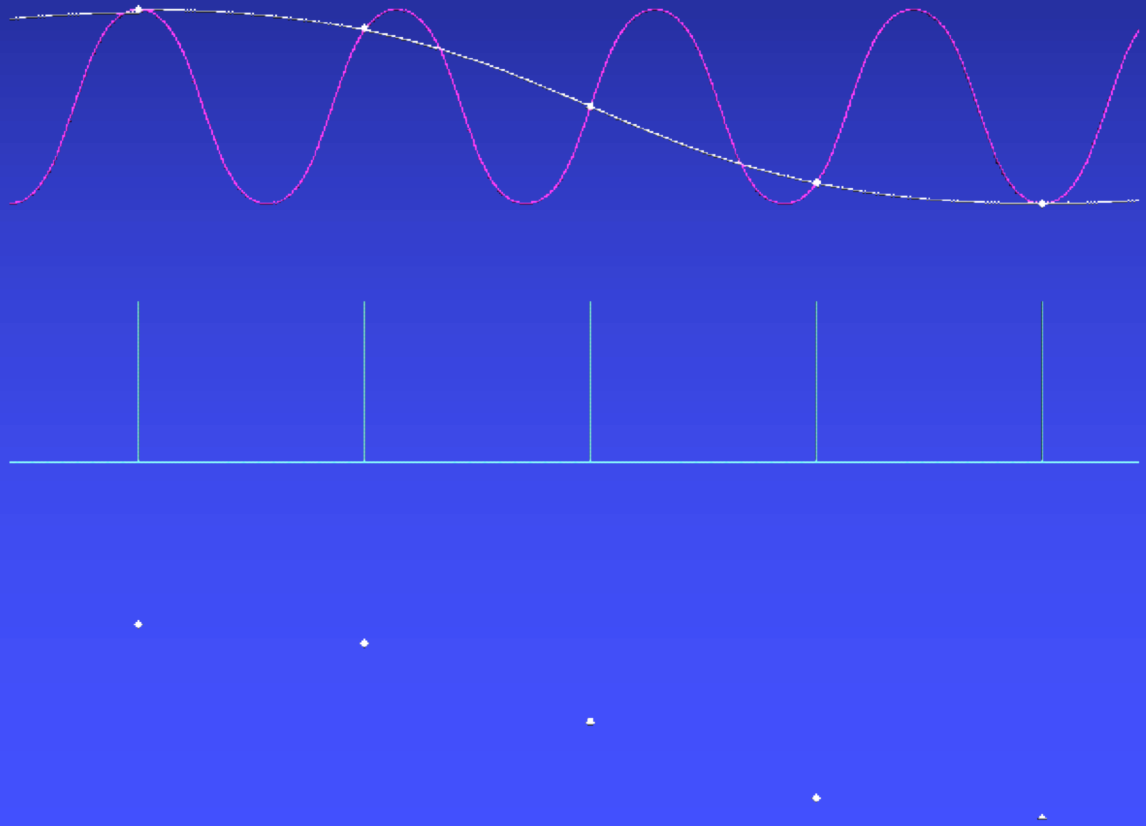
- Steps
  - Ideal continuous image
    - Filter
  - Filtered continuous image
    - Sample
  - Sampled image pixels
    - Reconstruction filter
  - Continuous displayed result

# Filtering, Sampling, Reconstruction

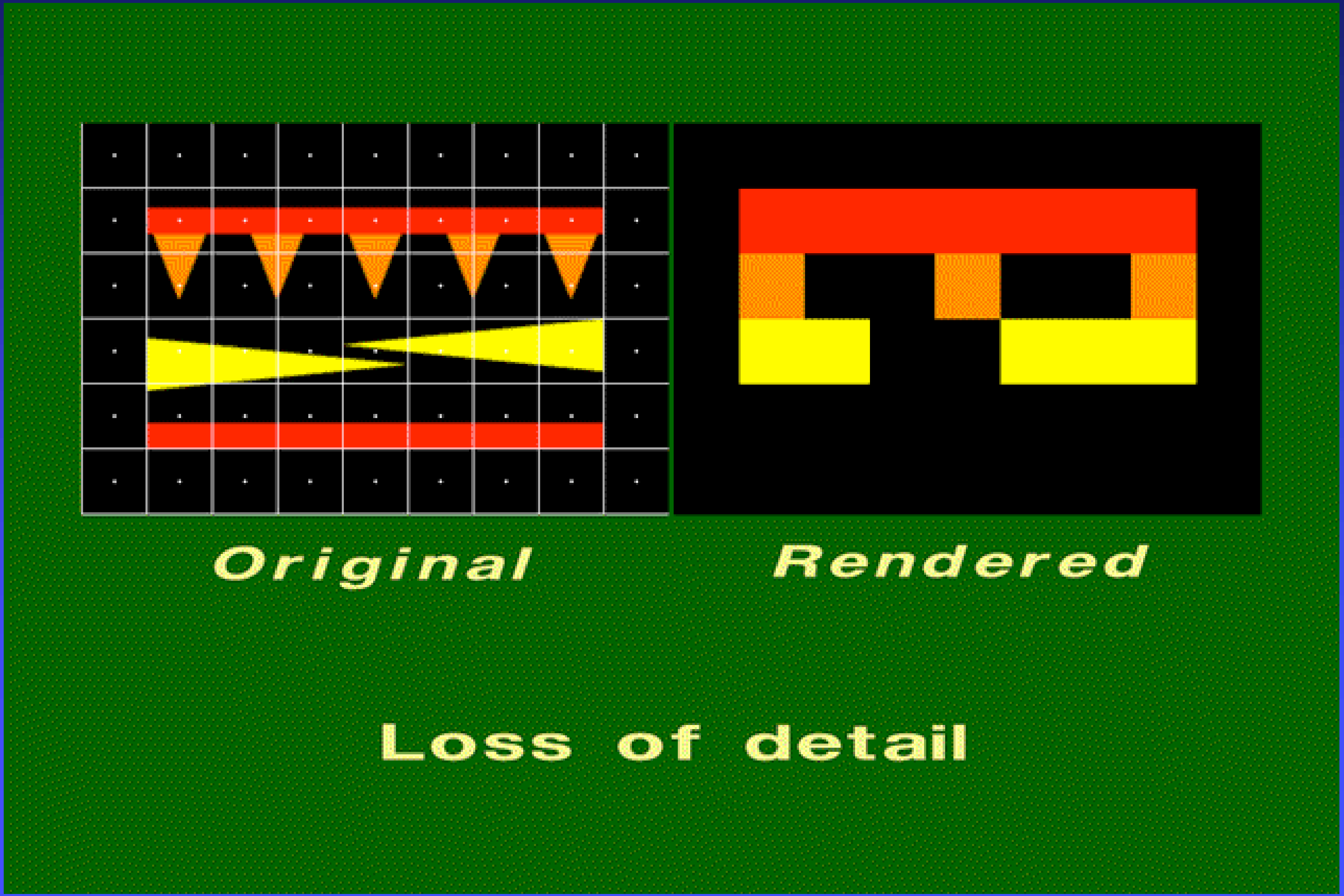
- Combine filter and sample
  - Ideal continuous image
    - Sampling filter
  - Sampled image pixels
    - Reconstruction filter
  - Continuous displayed result

# Aliasing

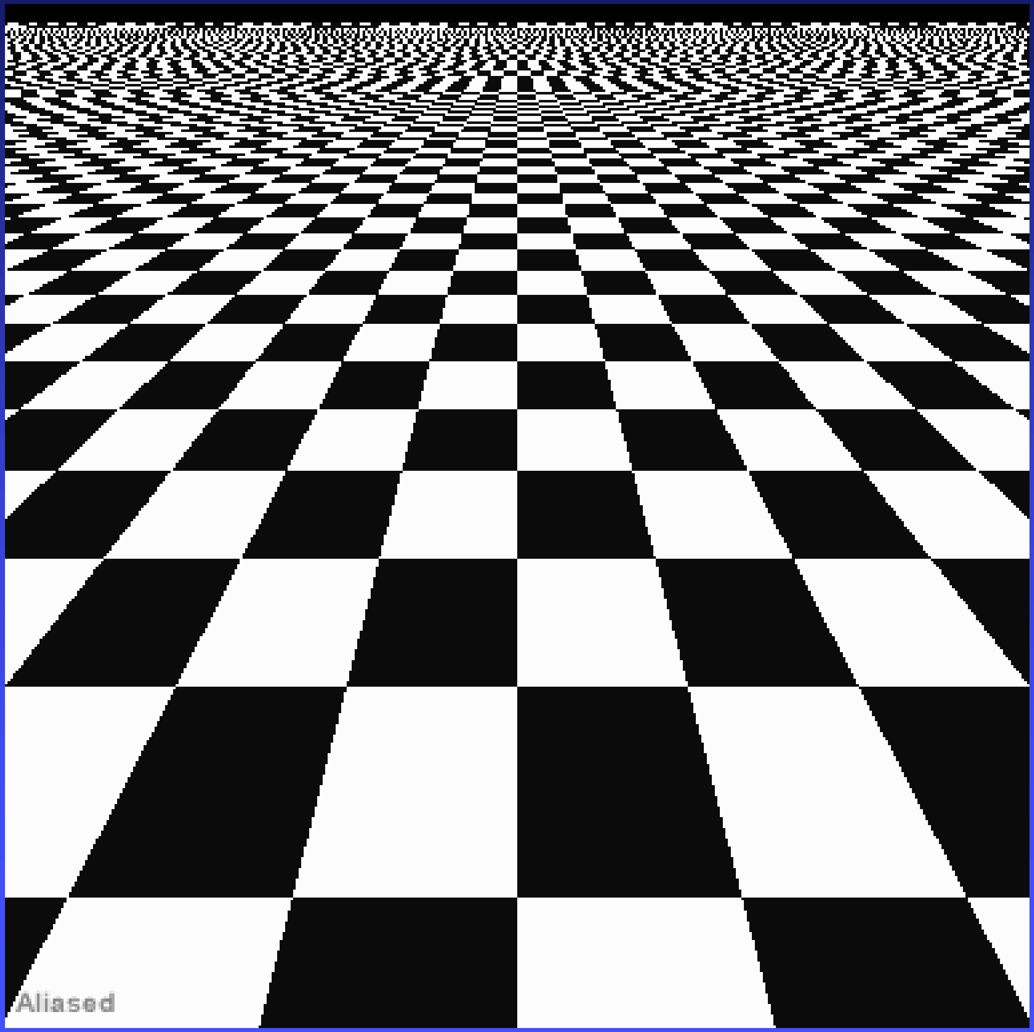
- High frequencies alias as low frequencies



# Aliasing in images



# Aliasing in animation

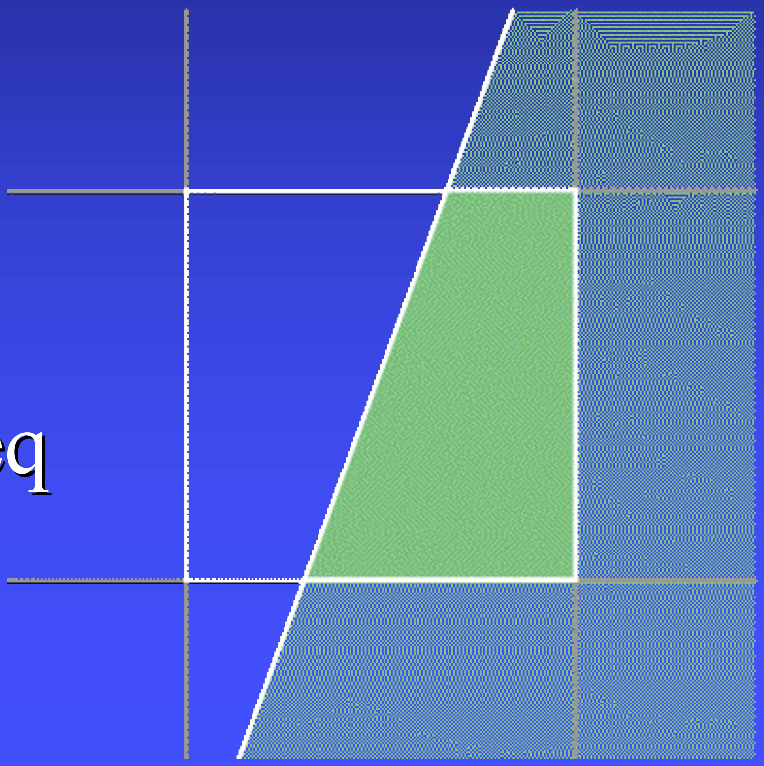


# Antialiasing

- Blur away frequencies that would alias
- Blur preferable to aliasing
- Can combine filtering and sampling
  - Evaluate convolution at sample points
- Filter kernel size
  - IIR = infinite impulse response
  - FIR = finite impulse response

# Analytic Area Sampling

- Compute "area" of pixel covered
- Box in spatial domain
  - Nice finite kernel
    - easy to compute
  - sinc in freq domain
    - Plenty of high freq
    - still aliases



# Analytic higher order filtering

- Fold better filter into rasterization
  - Can make rasterization much harder
  - Usually just done for lines
    - Draw with filter kernel “paintbrush”
- Only practical for finite filters

# Supersampling

- Numeric integration of filter
- Grid with equal weight = box filter
- Other filters:
  - Grid with unequal weights
  - Priority sampling
- Push up Nyquist frequency
  - Edges: ∞ frequency, still alias

# Adaptive sampling

- Numerical integration with varying step
- More samples in high contrast areas
- Easy with ray tracing, harder for others
- Possible bias

# Stochastic sampling

- Monte-Carlo integration of filter
- Sample distribution
  - Poisson disk
  - Jittered grid
- Aliasing  Noise

# Which filter kernel?

- Finite impulse response
  - Finite filter
  - Windowed filter
- Positive everywhere?
- Ringing?

# Which filter kernel?

- Windowed sync
- Windowed Gaussian (~0 @ 6□)
- Box, tent, cubic

# Resampling

- Image samples
  - Reconstruction filter
- Continuous image
  - Sampling filter
- Image samples