Viewing

CMSC 435/634

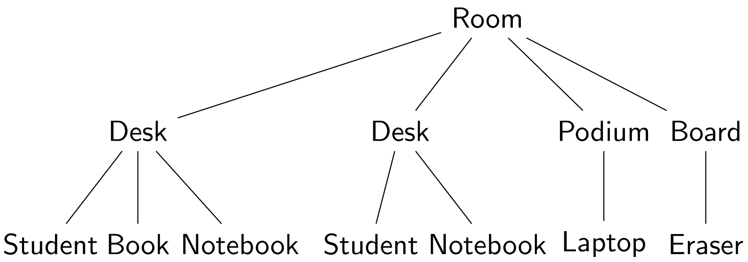
Coordinate System / Space

- Origin + Axes
- Reference frame
- Convert by matrix
  - $\vec{p}_{table} = TableFromPencil\ \vec{p}_{pencil}$
  - $\vec{p}_{room} = RoomFromTable\ TableFromPencil\ \vec{p}_{pencil}$
  - $\vec{p}_{room} = RoomFromPencil\ \vec{p}_{pencil}$

Spaces

- Object / Model
  - Logical coordinates for modeling
  - May have several more levels
- World
  - Common coordinates for everything
- View / Camera / Eye
  - eye/camera at (0,0,0), looking down Z (or -Z) axis
  - planes: left, right, top, bottom, near/hither, far/yon
- Normalized Device Coordinates (NDC) / Clip
  - Visible portion of scene from (-1,-1,-1) to (1,1,1)
  - Sometimes one or more components 0 to 1 instead of -1 to 1
- Raster / Pixel / Viewport
  - 0,0 to x-resolution, y-resolution
- Device / Screen
  - May translate to fit actual screen

Nesting


Room
Desk
Desk
Podium
Board
Student
Book
Notebook
Student
Notebook
Laptop
Eraser

Matrix Stack

- Remember transformation, return to it later
- Push a copy, modify the copy, pop
- Keep matrix and update matrix and inverse
- Push and pop both matrix and inverse together

```
transform(WorldFromRoom);
push;
transform(RoomFromDesk);
push;
transform(DeskFromStudent);
pop;  push;
transform(DeskFromBook);
...
```

Model→World / Model→View

- Model→World
  - All shading and rendering in World space
  - Transform all objects and lights
- Ray tracing implicitly does World→Raster
- Model→View
  - Serves just as well for single view

World→View

- Also called Viewing or Camera transform
- LookAt
  - $\overrightarrow{from}, \overrightarrow{to}, \overrightarrow{up}$
  - $\left[\begin{array}{c|c|c|c} \vec{u} & \vec{v} & \vec{w} & \overrightarrow{from} \end{array}\right]$
- Roll / Pitch / Yaw
  - Translate to camera center, rotate around camera
  - $R_z \; R_x \; R_y \; T$
  - Can have gimbal lock
- Orbit
  - Rotate around object center, translate out
  - $T \; R_z \; R_x \; R_y$
  - Also can have gimbal lock

View→NDC

- Also called *Projection* transform
- Orthographic / Parallel
  - Translate & Scale to view volume
  - $\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- Perspective
  - More complicated...

NDC→Raster

- Also called *Viewport* transform
- $[-1,1],[-1,1],[-1,1] \rightarrow [-\frac{1}{2}, n_x - \frac{1}{2}],[-\frac{1}{2}, n_y - \frac{1}{2}],[-\frac{1}{2}, n_z - \frac{1}{2}]$

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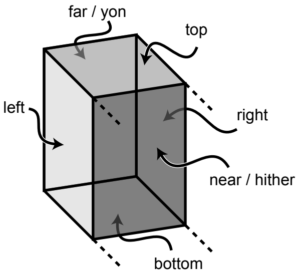
$$\begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & \frac{n_z}{2} & \frac{n_z-1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Raster$\rightarrow$Screen

- Usually just a translation
  - More complicated for tiled displays, domes, etc.
- Usually handled by windowing system

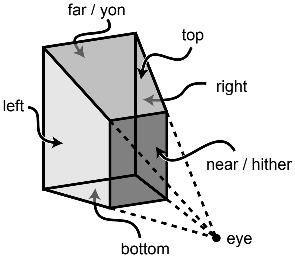
Perspective View Frustum

- Orthographic view volume is a rectangular volume



Perspective View Frustum

- Orthographic view volume is a rectangular volume
- Perspective is a truncated pyramid or *frustum*



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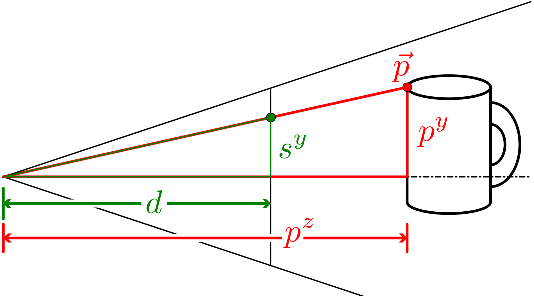
  - Use similar triangles



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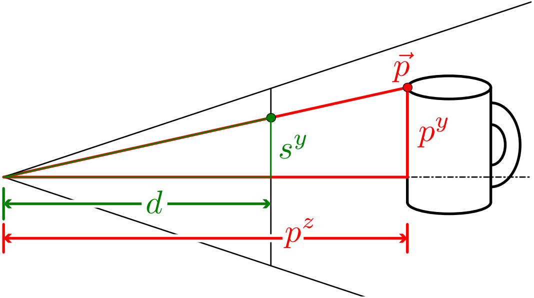
  - $s^y/d = p^y/p^z$



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- Perspective Transform
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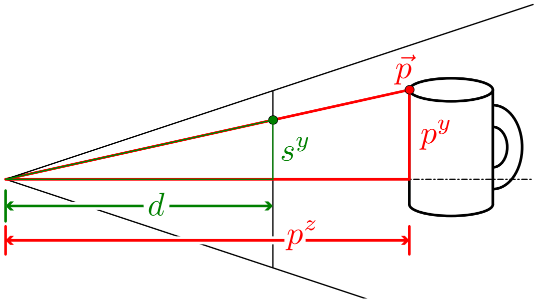
  - $s^y/d = p^y/p^z$ So $s^y = dp^y/p^z$



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Homogeneous Coordinates

- Rather than $(x, y, z, 1)$, use (x, y, z, w)
- Real 3D point is $(X, Y, Z) = (x/w, y/w, z/w)$
- Can represent Perspective Transform as 4x4 matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} p^x \\ p^y \\ p^z \\ 1 \end{bmatrix} = \begin{bmatrix} p^x \\ p^y \\ p^z \\ p^z/d \end{bmatrix} \rightarrow \begin{bmatrix} d\, p^x/p^z \\ d\, p^y/p^z \\ d \end{bmatrix}$$

Homogeneous Depth

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} p^x \\ p^y \\ p^z \\ 1 \end{bmatrix} = \begin{bmatrix} p^x \\ p^y \\ p^z \\ p^z/d \end{bmatrix} \rightarrow \begin{bmatrix} d\, p^x/p^z \\ d\, p^y/p^z \\ d \end{bmatrix}$$

- Lose depth information
- Can't get $d\, p'^z/p^z = p^z$
  - Plus $x/z, y/z, z$ isn't linear
- Use *Projective Geometry*

Projective Geometry

- If x, y, z lie on a plane, $x/z, y/z, 1/z$ also lie on a plane
- $1/z$ is strictly ordered: if $z_1 < z_2$, then $1/z_1 > 1/z_2$
- New matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p^x \\ p^y \\ p^z \\ 1 \end{bmatrix} = \begin{bmatrix} p^x \\ p^y \\ 1 \\ p^z \end{bmatrix} \rightarrow \begin{bmatrix} p^x/p^z \\ p^y/p^z \\ 1/p^z \end{bmatrix}$$

Getting Fancy

- Add scale & translate
  - Field of view
  - near/far range

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & c \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} p^x \\ p^y \\ p^z \\ 1 \end{bmatrix} = \begin{bmatrix} a\,p^x \\ a\,p^y \\ b\,p^z + c \\ -p^z \end{bmatrix} \to \begin{bmatrix} -a\,p^x/p^z \\ -a\,p^y/p^z \\ -b - c/p^z \end{bmatrix}$$

- $a = cotan(fieldOfView/2)$
- Solve for $n \to -1$ and $f \to 1$

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- $a = cotan(fieldOfView/2)$
- Solve for $n \rightarrow -1$ and $f \rightarrow 1$
  - $b = (n+f)/(n-f)$
  - $c = (2\,n\,f)/(f-n)$

On Field of View

- Given image dimensions, set distance
  - Camera image sensor and focal length
- Given field of view angle in square window
- Non-square aspect ratio
  - Given horizontal (or vertical) field of view
  - Given diagonal field of view
- Off-center projection
  - Tiled displays
  - Head tracking