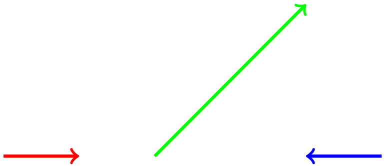
Linear Algebra Review

CMSC 435/634

Abstract Vectors

($\vec{u}$, $\vec{v}$, $\vec{w}$ vectors; a, b, c scalars)



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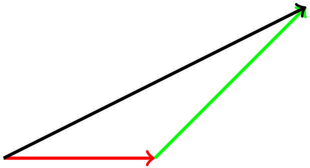

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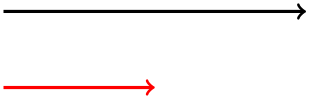

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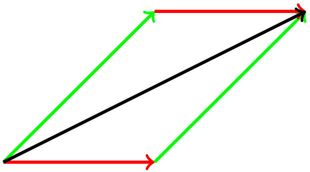

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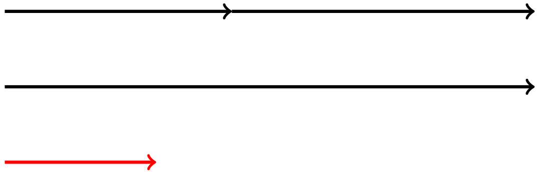

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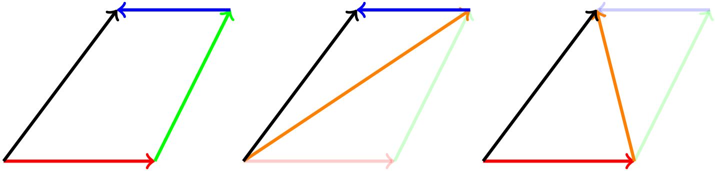
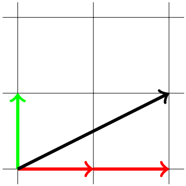


Basis Vectors

Vector as linear combination of *basis vectors*

- $\vec{v} = 2\hat{i} + 1\hat{j} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$



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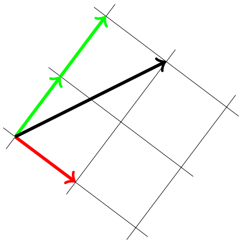

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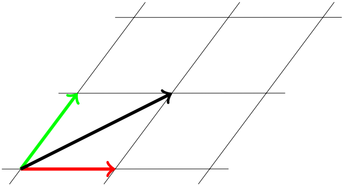

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- I like columns for points/vectors, rows for normals

Matrices

- Matrix: $A = \begin{bmatrix} a_0^0 & a_1^0 \\ a_0^1 & a_1^1 \end{bmatrix} = \left[a_j^i \right] = \left[a_{row}^{column} \right]$

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- $a_\alpha^i b_j^\alpha$ is *Einstein Summation Notation*
  - Loop and sum over matching index variables

Einstein Summation and Code

- Math: $c^i_j = a^i_\alpha b^\alpha_j$
- Code:

```
for(int i=0; i<N; ++i) {
   for(int j=0; j<M; ++j) {
      c[i][j] = 0;
      for(int α=0; α<K; α++) {
         c[i][j] = c[i][j] + a[i][α] * b[α][j];
      }
   }
}
```

Adjoint and Inverse

- Inverse: $A^{-1}A = AA^{-1} = I$
- Determinant: $|A|$
  - $|a| = a$
  - $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a|d| - b|c|$
  - $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$
- Adjoint: $A^* = cof(A)^T$ (matrix of cofactors $cof(A)$)
- $A^{-1} = \frac{A^*}{|A|}$

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  - $\vec{v} \bullet \vec{v} = 0 \leftrightarrow \vec{v} = \vec{0}$
- Equivalent notations
  - Vector: $\vec{u} \bullet \vec{v}$
  - Matrix: $U^T V$
  - Summation: $u_\alpha v^\alpha$

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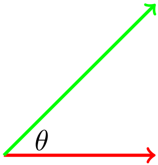

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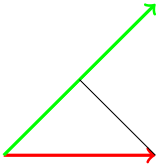

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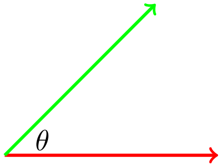
  - If $|\vec{u}| = |\vec{v}| = 1$, gives just $\cos\theta$



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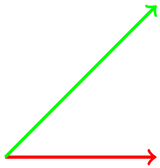
- *Orthonormal*: set of vectors both orthogonal and normal
- *Orthogonal matrix*: rows (& columns) **orthonormal**
  - For orthogonal matrices, $A^{-1} = A^T$

3D Cross Product

$\vec{u} \times \vec{v}$



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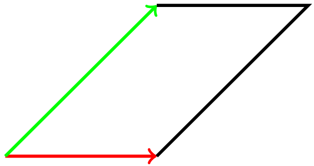

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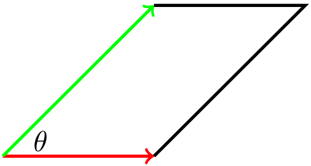

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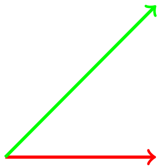

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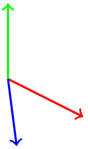
- $\begin{bmatrix} w^0 \\ w^1 \\ w^2 \end{bmatrix} = \begin{bmatrix} u^1 v^2 - u^2 v^1 \\ u^2 v^0 - u^0 v^2 \\ u^0 v^1 - u^1 v^0 \end{bmatrix}$

Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$



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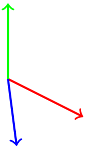
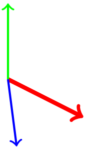
Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)



Building an Orthogonal Basis



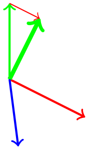
Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$

Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

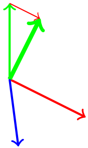
- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \hat{u}' \left(\vec{v} \bullet \hat{u}'\right)$



Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

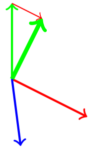
- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \frac{\vec{u'}}{|\vec{u'}|}\left(\vec{v} \bullet \frac{\vec{u'}}{|\vec{u'}|}\right)$



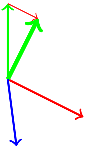
Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / |\vec{u'}|^2$



Building an Orthogonal Basis



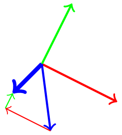
Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'} \; \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$

Building an Orthogonal Basis

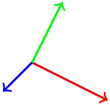
Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$



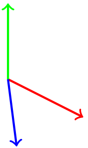
Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$



- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$

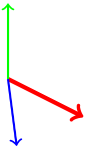
Building an Orthogonal Basis



Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$
- Cross-product (3D only)

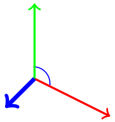
Building an Orthogonal Basis



Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$
- Cross-product (3D only)
  - $\vec{u'} = \vec{u}$

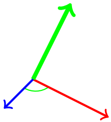
Building an Orthogonal Basis



Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$
- Cross-product (3D only)
  - $\vec{u'} = \vec{u}$
  - $\vec{w'} = \vec{u'} \times \vec{v}$

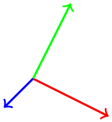
Building an Orthogonal Basis



Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u}' = \vec{u}$
  - $\vec{v}' = \vec{v} - \vec{u}' \ \vec{v} \bullet \vec{u}' / \vec{u}' \bullet \vec{u}'$
  - $\vec{w}' = \vec{w} - \vec{u}' \ \vec{w} \bullet \vec{u}' / \vec{u}' \bullet \vec{u}' - \vec{v}' \ \vec{w} \bullet \vec{v}' / \vec{v}' \bullet \vec{v}'$
- Cross-product (3D only)
  - $\vec{u}' = \vec{u}$
  - $\vec{w}' = \vec{u}' \times \vec{v}$
  - $\vec{v}' = \vec{w}' \times \vec{u}'$

Building an Orthogonal Basis



Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$
- Cross-product (3D only)
  - $\vec{u'} = \vec{u}$
  - $\vec{w'} = \vec{u'} \times \vec{v}$
  - $\vec{v'} = \vec{w'} \times \vec{u'}$