3D Transformations

CMSC 435/634

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Transformation

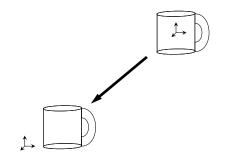
Webster: The operation of changing one configuration or expression into another in accordance with a mathematical rule



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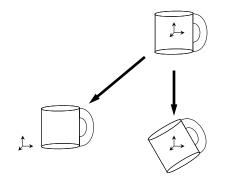
Transformation

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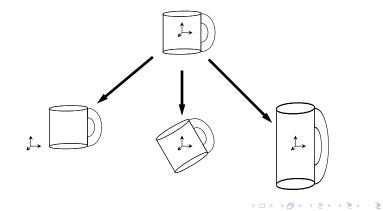
Transformation

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-Generic Transforms

Using Transformation

Points on object represented as vector offset from origin

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- ► Transform is a vector to vector function ► $\vec{p}' = f(\vec{p})$
- Relativity:

• Inverse transform, $\vec{p} = f^{-1}(\vec{p}')$



-Generic Transforms

Using Transformation

Points on object represented as vector offset from origin

- Transform is a vector to vector function
 - ▶ $\vec{p}' = f(\vec{p})$
- ► Relativity:
 - > From $ec{p}'$ point of view, object is transformed
- Inverse transform, $\vec{p} = f^{-1}(\vec{p}')$



-Generic Transforms

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- Transform is a vector to vector function

► $\vec{p}' = f(\vec{p})$

- ► Relativity:
 - From p' point of view, object is transformed
 - \sim From $ec{p}$ point of view, coordinate system changes

• Inverse transform, $\vec{p} = f^{-1}(\vec{p}')$



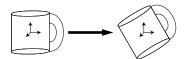
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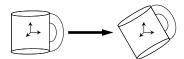
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- Generic Transforms

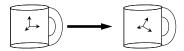
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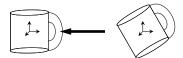
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-Generic Transforms

Composing Transforms

Order matters

 \blacktriangleright $R(T(\vec{p})) = R \circ T(\vec{p})$

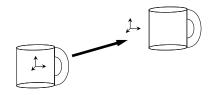
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-Generic Transforms

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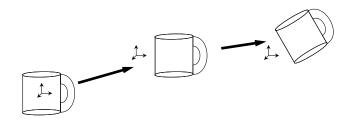


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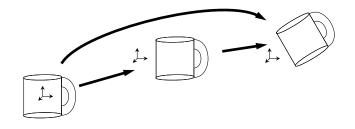
-Generic Transforms

Composing Transforms

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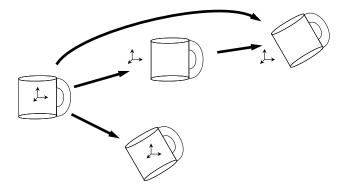


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-Generic Transforms

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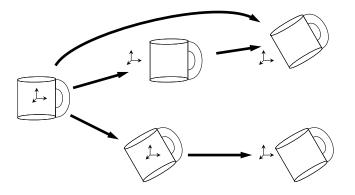
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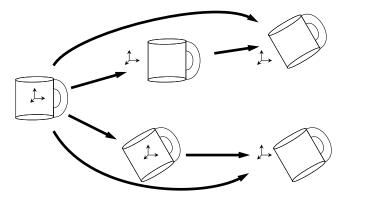
-Generic Transforms

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-Generic Transforms

Inverting Composed Transforms

▶ $(R \circ T)^{-1}(\vec{p}')$

Reverse order

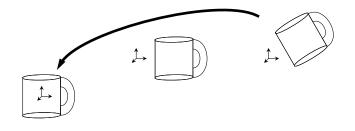
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-Generic Transforms

Inverting Composed Transforms

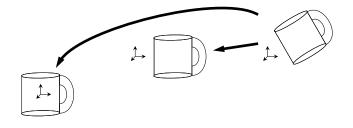
- Reverse order
 - $(R \circ T)^{-1}(\vec{p}')$



-Generic Transforms

Inverting Composed Transforms

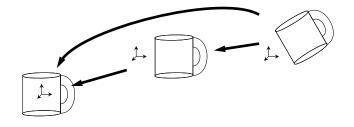
- Reverse order
 - $(R \circ T)^{-1}(\vec{p}') = T^{-1}(R^{-1}(\vec{p}'))$



-Generic Transforms

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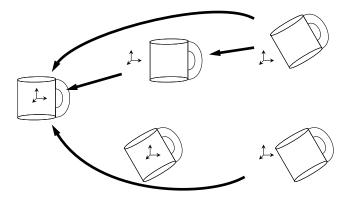


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-Generic Transforms

Inverting Composed Transforms

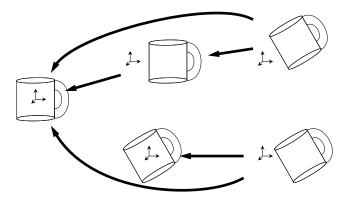
- Reverse order
 - $(R \circ T)^{-1}(\vec{p}') = T^{-1}(R^{-1}(\vec{p}'))$ • $(T \circ R)^{-1}(\vec{p}')$



-Generic Transforms

Inverting Composed Transforms

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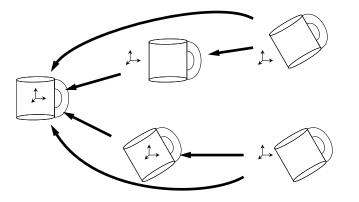
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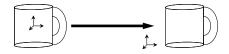
- Common Transforms

Translation

$$\vec{p}' = \vec{p} + \vec{t} \begin{bmatrix} p'^{x} \\ p'^{y} \\ p'^{z} \end{bmatrix} = \begin{bmatrix} p^{x} \\ p^{y} \\ p^{z} \end{bmatrix} + \begin{bmatrix} t^{x} \\ t^{y} \\ t^{z} \end{bmatrix} = \begin{bmatrix} p^{x} + t^{x} \\ p^{y} + t^{y} \\ p^{z} + t^{z} \end{bmatrix}$$

▶ \vec{t} says where \vec{p} -space origin ends up $(\vec{p}' = \vec{0} + \vec{t})$

• Composition: $\vec{p}' = (\vec{p} + \vec{t_0}) + \vec{t_1} = \vec{p} + (\vec{t_0} + \vec{t_1})$



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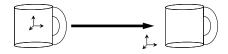
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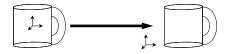
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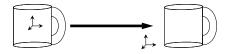
- Common Transforms

Translation

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Common Transforms

Linear Transforms

Linear Transforms

$$\blacktriangleright \begin{bmatrix} p'^{x} \\ p'^{y} \\ p'^{z} \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} p^{x} \\ p^{y} \\ p^{z} \end{bmatrix}$$

• Matrix says where \vec{p} -space axes end up

> Composition: $\vec{p}' = M$ (N \vec{p}) = (M N) \vec{p}

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Common Transforms

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- Common Transforms

Linear Transforms

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• Matrix says where \vec{p} -space axes end up

•
$$\begin{bmatrix} a \\ d \\ g \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

• Composition $p' = M(N, p) = (M, N)$

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- Common Transforms

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$$\begin{bmatrix} b \\ e \\ h \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

• Composition: $\vec{p}' = M (N \vec{p}) = (M N)\vec{p}$

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• Matrix says where \vec{p} -space axes end up

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$$\begin{bmatrix} c \\ f \\ i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

• Composition: $\vec{p}' = M (N \vec{p}) = (M N)\vec{p}$

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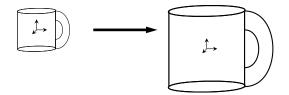
Common Transforms

Linear Transforms

Common case: Scaling

$$\begin{bmatrix} p'^{x} \\ p'^{y} \\ p'^{z} \end{bmatrix} = \begin{bmatrix} s_{x} p^{x} \\ s_{y} p^{y} \\ s_{z} p^{z} \end{bmatrix} = \begin{bmatrix} s_{x} & 0 & 0 \\ 0 & s_{y} & 0 \\ 0 & 0 & s_{z} \end{bmatrix} \begin{bmatrix} p^{x} \\ p^{y} \\ p^{z} \end{bmatrix}$$

$$\models \text{ Inverse:} \begin{bmatrix} 1/s_{x} & 0 & 0 \\ 0 & 1/s_{y} & 0 \\ 0 & 0 & 1/s_{z} \end{bmatrix}$$



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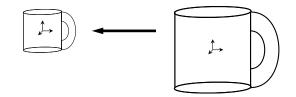
Common Transforms

Linear Transforms

Common case: Scaling

$$\begin{bmatrix} p'^{x} \\ p'^{y} \\ p'^{z} \end{bmatrix} = \begin{bmatrix} s_{x} p^{x} \\ s_{y} p^{y} \\ s_{z} p^{z} \end{bmatrix} = \begin{bmatrix} s_{x} & 0 & 0 \\ 0 & s_{y} & 0 \\ 0 & 0 & s_{z} \end{bmatrix} \begin{bmatrix} p^{x} \\ p^{y} \\ p^{z} \end{bmatrix}$$

$$\models \text{ Inverse:} \begin{bmatrix} 1/s_{x} & 0 & 0 \\ 0 & 1/s_{y} & 0 \\ 0 & 0 & 1/s_{z} \end{bmatrix}$$



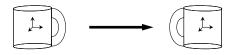
Common Transforms

Linear Transforms

Common case: Reflection

Negative scaling

$$\blacktriangleright \begin{bmatrix} p'^{x} \\ p'^{y} \\ p'^{z} \end{bmatrix} = \begin{bmatrix} -p^{x} \\ p^{y} \\ p^{z} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p^{x} \\ p^{y} \\ p^{z} \end{bmatrix}$$

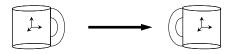


Common Transforms

Linear Transforms

Common case: Reflection

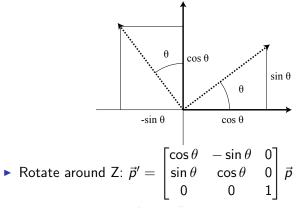
- Negative scaling
- $\begin{bmatrix} p'^{x} \\ p'^{y} \\ p'^{z} \end{bmatrix} = \begin{bmatrix} -p^{x} \\ p^{y} \\ p^{z} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p^{x} \\ p^{y} \\ p^{z} \end{bmatrix}$



Common Transforms

Linear Transforms

Common case: Rotation



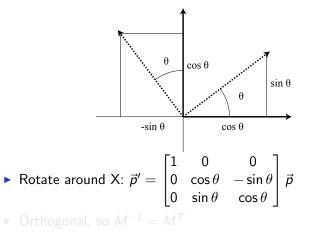
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• Orthogonal, so $M^{-1} = M^{-1}$

Common Transforms

Linear Transforms

Common case: Rotation

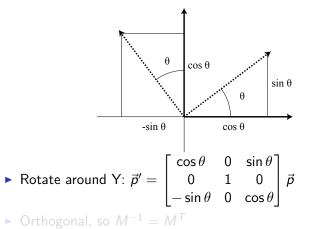


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Common Transforms

Linear Transforms

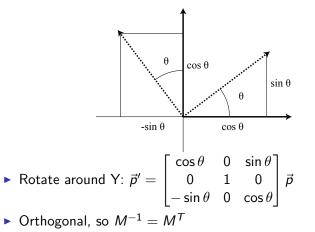
Common case: Rotation



Common Transforms

Linear Transforms

Common case: Rotation



Common Transforms

Linear Transforms

Composing Transforms

- Scale by s along axis a
 - ▶ Rotate to align *ā* with Z

- ► Scale along Z
- Rotate back

Common Transforms

Linear Transforms

Composing Transforms

- Scale by s along axis a
 - Rotate to align \vec{a} with Z

- ► Scale along Z
- Rotate back

Common Transforms

Linear Transforms

Composing Transforms

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Common Transforms

Linear Transforms

Composing Transforms

- Scale by s along axis a
 - Rotate to align \vec{a} with Z

- Scale along Z
- Rotate back

Common Transforms

Rotate by α around X into XZ plane

• Projection of \vec{a} onto YZ: $\begin{bmatrix} 0 \\ a^y \\ a^z \end{bmatrix}$

length $d = \sqrt{(a^y)^2 + (a^z)^2}$ So $\cos \alpha = a^z/d$, $\sin \alpha = a^y/d$ $R_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a^z/d & -a^y/d \\ 0 & a^y/d & a^z/d \end{bmatrix}$ Result $\vec{a}' = \begin{bmatrix} a^x \\ 0 \\ d \end{bmatrix}$

Common Transforms

Rotate by α around X into XZ plane

Projection of
$$\vec{a}$$
 onto YZ:
$$\begin{bmatrix} 0\\a^{y}\\a^{z} \end{bmatrix}$$
length $d = \sqrt{(a^{y})^{2} + (a^{z})^{2}}$
So $\cos \alpha = a^{z}/d$, $\sin \alpha = a^{y}/d$
 $R_{X} = \begin{bmatrix} 1 & 0 & 0\\ 0 & a^{z}/d & -a^{y}/d\\ 0 & a^{y}/d & a^{z}/d \end{bmatrix}$
Result $\vec{a}' = \begin{bmatrix} a^{x}\\ 0\\ d \end{bmatrix}$

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Common Transforms

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Common Transforms

Rotate by α around X into XZ plane

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Common Transforms

Rotate by α around X into XZ plane

Projection of
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 onto YZ:
$$\begin{bmatrix} 0\\ a^{y}\\ a^{z} \end{bmatrix}$$
length $d = \sqrt{(a^{y})^{2} + (a^{z})^{2}}$
So $\cos \alpha = a^{z}/d$, $\sin \alpha = a^{y}/d$
 $R_{X} = \begin{bmatrix} 1 & 0 & 0\\ 0 & a^{z}/d & -a^{y}/d\\ 0 & a^{y}/d & a^{z}/d \end{bmatrix}$
Result $\vec{a}' = \begin{bmatrix} a^{x}\\ 0\\ d \end{bmatrix}$

Common Transforms

Linear Transforms

Rotate by β around Y to Z axis

$$\vec{a}' = \begin{bmatrix} a^{x} \\ 0 \\ d \end{bmatrix}$$

$$\models \text{ length } = 1$$

$$\models \text{ So } \cos\beta = d, \sin\beta = a$$

$$\models R_{Y} = \begin{bmatrix} d & 0 & -a^{x} \\ 0 & 1 & 0 \\ a^{x} & 0 & d \end{bmatrix}$$

$$\models \text{ Result } \vec{a}'' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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Common Transforms

Linear Transforms

Rotate by β around Y to Z axis

$$\vec{a}' = \begin{bmatrix} a^{x} \\ 0 \\ d \end{bmatrix}$$

$$\mathbf{i} \text{ length} = 1$$

$$\mathbf{i} \text{ So } \cos \beta = d, \sin \beta = a$$

$$\mathbf{k}_{Y} = \begin{bmatrix} d & 0 & -a^{x} \\ 0 & 1 & 0 \\ a^{x} & 0 & d \end{bmatrix}$$

$$\mathbf{k} \text{ Result } \vec{a}'' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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Common Transforms

Linear Transforms

Rotate by β around Y to Z axis

 $\blacktriangleright \vec{a}' = \begin{bmatrix} a^x \\ 0 \\ d \end{bmatrix}$ \blacktriangleright length = 1 • So $\cos \beta = d$, $\sin \beta = a^x$ $\triangleright R_Y = \begin{bmatrix} d & 0 & -a^x \\ 0 & 1 & 0 \\ a^x & 0 & d \end{bmatrix}$

Common Transforms

Linear Transforms

Rotate by β around Y to Z axis

 $\bullet \ \vec{a}' = \begin{bmatrix} a^x \\ 0 \\ d \end{bmatrix}$ \blacktriangleright length = 1 • So $\cos \beta = d$, $\sin \beta = a^x$ $\blacktriangleright R_Y = \begin{bmatrix} d & 0 & -a^x \\ 0 & 1 & 0 \\ a^x & 0 & d \end{bmatrix}$ • Result $\vec{a}'' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Common Transforms

Linear Transforms

Rotate by β around Y to Z axis

 $\bullet \vec{a}' = \begin{bmatrix} a^x \\ 0 \\ a \end{bmatrix}$ \blacktriangleright length = 1 • So $\cos \beta = d$, $\sin \beta = a^x$ $\blacktriangleright R_Y = \begin{bmatrix} d & 0 & -a^x \\ 0 & 1 & 0 \\ a^x & 0 & d \end{bmatrix}$ • Result $\vec{a}'' = \begin{bmatrix} 0\\0\\1 \end{bmatrix}$

Common Transforms

Linear Transforms

Composing Transforms

• Scale by *s* along Z:
$$S_Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix}$$

Scale by *s* along axis *ā*

- Rotate to align a with Z
- Scale along Z
- Rotate back
- ► S_ZR_YR_X p

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Common Transforms

Linear Transforms

Composing Transforms

Scale by *s* along Z:
$$S_Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix}$$

- Scale by s along axis a
 - Rotate to align \vec{a} with Z
 - Scale along Z
 - Rotate back

$$\vec{p}' = R_X^{-1} R_Y^{-1} S_Z R_Y R_X \vec{p}$$

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Common Transforms

Linear Transforms

Composing Transforms

• Scale by *s* along Z:
$$S_Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix}$$

- Scale by s along axis a
 - Rotate to align \vec{a} with Z
 - Scale along Z
 - Rotate back
 - $\vec{p}' = R_X^{-1} R_Y^{-1} S_Z R_Y R_X \vec{p}$

Common Transforms

Linear Transforms

Composing Transforms

Scale by *s* along Z:
$$S_Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix}$$

- Scale by s along axis a
 - Rotate to align \vec{a} with Z
 - Scale along Z
 - Rotate back

•
$$\vec{p}' = R_X^{-1} R_Y^{-1} S_Z R_Y R_X \vec{p}$$

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Affine Transforms

Affine Transforms

- Affine = Linear + Translation
- Composition? A $(B \ \vec{p} + \vec{t_0}) + \vec{t_1} = A \ B \ \vec{p} + A \ \vec{t_0} + \vec{t_1}$

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Yuck!

-Affine Transforms

Affine Transforms

- Affine = Linear + Translation
- Composition? A $(B \vec{p} + \vec{t_0}) + \vec{t_1} = A B \vec{p} + A \vec{t_0} + \vec{t_1}$

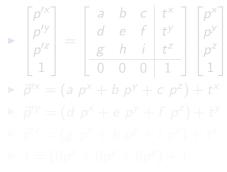
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Yuck!

Affine Transforms

Homogeneous Coordinates

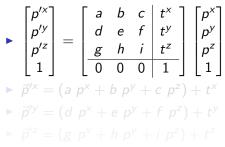
Add a '1' to each point



Affine Transforms

Homogeneous Coordinates

Add a '1' to each point

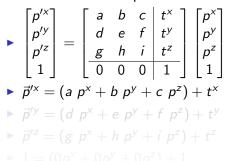


► $1 = (0p^{x} + 0p^{y} + 0p^{z}) + 1$

Affine Transforms

Homogeneous Coordinates

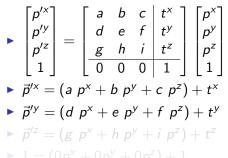
Add a '1' to each point



Affine Transforms

Homogeneous Coordinates

Add a '1' to each point



Affine Transforms

Homogeneous Coordinates

Add a '1' to each point

$$\begin{bmatrix} p'^{x} \\ p'^{y} \\ p'^{z} \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c & t^{x} \\ d & e & f & t^{y} \\ g & h & i & t^{z} \\ \hline 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p^{x} \\ p^{y} \\ p^{z} \\ 1 \end{bmatrix}$$

$$\vec{p}'^{x} = (a p^{x} + b p^{y} + c p^{z}) + t^{x}$$

$$\vec{p}'^{y} = (d p^{x} + e p^{y} + f p^{z}) + t^{y}$$

$$\vec{p}'^{z} = (g p^{x} + h p^{y} + i p^{z}) + t^{z}$$

$$1 = (0p^{x} + 0p^{y} + 0p^{z}) + 1$$

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- Affine Transforms

Homogeneous Coordinates

Add a '1' to each point

$$\begin{bmatrix} p'^{x} \\ p'^{y} \\ p'^{z} \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c & t^{x} \\ d & e & f & t^{y} \\ g & h & i & t^{z} \\ \hline 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p^{x} \\ p^{y} \\ p^{z} \\ 1 \end{bmatrix}$$

$$\vec{p}'^{x} = (a p^{x} + b p^{y} + c p^{z}) + t^{x}$$

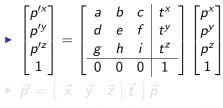
$$\vec{p}'^{y} = (d p^{x} + e p^{y} + f p^{z}) + t^{y}$$

$$\vec{p}'^{z} = (g p^{x} + h p^{y} + i p^{z}) + t^{z}$$

$$1 = (0p^{x} + 0p^{y} + 0p^{z}) + 1$$

-Affine Transforms

Homogeneous Coordinates

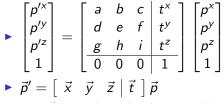


t says where the p-space origin ends up
 X. V. Z say where the p-space axes end up

Composition: Just matrix multiplies!

Affine Transforms

Homogeneous Coordinates



- \vec{t} says where the \vec{p} -space origin ends up
- \vec{x} , \vec{y} , \vec{z} say where the \vec{p} -space axes end up

Composition: Just matrix multiplies!

- Affine Transforms

Homogeneous Coordinates

$$\begin{array}{c} \left[\begin{array}{c} p'^{x} \\ p'^{y} \\ p'^{z} \\ 1 \end{array} \right] = \left[\begin{array}{c} a & b & c & t^{x} \\ d & e & f & t^{y} \\ \hline g & h & i & t^{z} \\ \hline 0 & 0 & 0 & 1 \end{array} \right] \left[\begin{array}{c} p^{x} \\ p^{y} \\ p^{z} \\ 1 \end{array} \right] \\ \bullet \vec{p}' = \left[\begin{array}{c} \vec{x} & \vec{y} & \vec{z} & | \vec{t} \end{array} \right] \vec{p} \\ \bullet \vec{t} \text{ says where the } \vec{p} \text{-space origin ends up} \end{array}$$

 \blacktriangleright \vec{x} , \vec{y} , \vec{z} say where the \vec{p} -space axes end up

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Composition: Just matrix multiplies!

-Affine Transforms

Homogeneous Coordinates

$$\vec{p}^{\prime x}_{p^{\prime y}_{l}} = \begin{bmatrix} a & b & c & t^{x} \\ d & e & f & t^{y} \\ p^{\prime z}_{l} \end{bmatrix} \begin{bmatrix} p^{x}_{l} \\ p^{y}_{l} \\ p^{z}_{l} \end{bmatrix}$$
$$\vec{p}^{\prime} = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} & | \vec{t} \end{bmatrix} \vec{p}$$

- \vec{t} says where the \vec{p} -space origin ends up
- \vec{x} , \vec{y} , \vec{z} say where the \vec{p} -space axes end up

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Composition: Just matrix multiplies!

-Affine Transforms

Homogeneous Coordinates

$$\vec{p}^{\prime x}_{p^{\prime y}_{1}} = \begin{bmatrix} a & b & c & t^{x} \\ d & e & f & t^{y} \\ p^{\prime z}_{1} \end{bmatrix} = \begin{bmatrix} a & b & c & t^{x} \\ d & e & f & t^{y} \\ \hline \frac{g & h & i & t^{z}}{0 & 0 & 0 & 1} \end{bmatrix} \begin{bmatrix} p^{x}_{p^{y}_{1}} \\ p^{y}_{1} \\ p^{z}_{1} \end{bmatrix}$$

$$\vec{p}^{\prime} = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} & | \vec{t} \end{bmatrix} \vec{p}$$

- \vec{t} says where the \vec{p} -space origin ends up
- \vec{x} , \vec{y} , \vec{z} say where the \vec{p} -space axes end up

Composition: Just matrix multiplies!

Affine Transforms

Composing Transforms

• Rotate by θ about line between \vec{p}_0 and \vec{p}_1 :

- Translate \vec{p}_0 to origin
- Rotate to align $\vec{p}_1 \vec{p_0}$ with Z
- Rotate by θ around Z
- Undo $\vec{p}_1 \vec{p_0}$ rotation
- Undo translation

$\blacktriangleright T^{-1}R_{\Sigma}^{-1}R_{\Sigma}^{-1}R_{Z}(\theta)R_{Y}R_{X}T$

Affine Transforms

Composing Transforms

• Rotate by θ about line between \vec{p}_0 and \vec{p}_1 :

- Translate \vec{p}_0 to origin
- Rotate to align $\vec{p}_1 \vec{p_0}$ with Z
- Rotate by θ around Z
- Undo $\vec{p}_1 \vec{p_0}$ rotation
- Undo translation
- $\succ T^{-1}R_X^{-1}R_Y^{-1}R_Z(\theta)R_YR_XT$

Affine Transforms

Composing Transforms

• Rotate by θ about line between \vec{p}_0 and \vec{p}_1 :

- Translate \vec{p}_0 to origin
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- Affine Transforms

Composing Transforms

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- Affine Transforms

Composing Transforms

• Rotate by θ about line between \vec{p}_0 and \vec{p}_1 :

- Translate \vec{p}_0 to origin
- Rotate to align $\vec{p}_1 \vec{p_0}$ with Z
- Rotate by θ around Z
- Undo $\vec{p}_1 \vec{p_0}$ rotation
- Undo translation
- $T^{-1}R_X^{-1}R_Y^{-1}R_Z(\theta)R_YR_XT$

-Vectors and Normals

Vectors

Transform by Jacobian Matrix

- Matrix of partial derivatives
 - $\vec{p}^{\prime x} = a p^{x} + b p^{y} + c p^{z} + t^{x}$ $\vec{p}^{\prime y} = d p^{x} + e p^{y} + f p^{z} + t^{y}$ $\vec{p}^{\prime z} = g p^{x} + h p^{y} + i p^{z} + t^{z}$

-Vectors and Normals

Vectors

- Transform by Jacobian Matrix
- Matrix of partial derivatives
 - $\blacktriangleright \begin{bmatrix} \vec{p}'^{x} \\ \vec{p}'^{y} \\ \vec{p}'^{z} \end{bmatrix} = \begin{bmatrix} a \ p^{x} + b \ p^{y} + c \ p^{z} + t^{x} \\ d \ p^{x} + e \ p^{y} + f \ p^{z} + t^{y} \\ g \ p^{x} + h \ p^{y} + i \ p^{z} + t^{z} \end{bmatrix}$

-Vectors and Normals

Vectors

- Transform by Jacobian Matrix
- Matrix of partial derivatives

$$\bullet \begin{bmatrix} \vec{p}'^{x} \\ \vec{p}'^{y} \\ \vec{p}'^{z} \end{bmatrix} = \begin{bmatrix} a \ p^{x} + b \ p^{y} + c \ p^{z} + t^{x} \\ d \ p^{x} + e \ p^{y} + f \ p^{z} + t^{y} \\ g \ p^{x} + h \ p^{y} + i \ p^{z} + t^{z} \end{bmatrix}$$

-Vectors and Normals

Vectors

- Transform by Jacobian Matrix
- Matrix of partial derivatives

$$\begin{bmatrix} \vec{p}'^{x} \\ \vec{p}'^{y} \\ \vec{p}'^{z} \end{bmatrix} = \begin{bmatrix} a \ p^{x} + b \ p^{y} + c \ p^{z} + t^{x} \\ d \ p^{x} + e \ p^{y} + f \ p^{z} + t^{y} \\ g \ p^{x} + h \ p^{y} + i \ p^{z} + t^{z} \end{bmatrix}$$

$$J = \begin{bmatrix} \partial p'^{x} / \partial p^{x} \ \partial p'^{x} / \partial p^{y} \ \partial p'^{x} / \partial p^{z} \\ \partial p'^{y} / \partial p^{x} \ \partial p'^{y} / \partial p^{y} \ \partial p'^{y} / \partial p^{z} \\ \partial p'^{z} / \partial p^{x} \ \partial p'^{z} / \partial p^{y} \ \partial p'^{z} / \partial p^{z} \end{bmatrix}$$

-Vectors and Normals

Vectors

- Transform by Jacobian Matrix
- Matrix of partial derivatives

$$\begin{bmatrix} \vec{p}'^{x} \\ \vec{p}'^{y} \\ \vec{p}'^{z} \end{bmatrix} = \begin{bmatrix} a \ p^{x} + b \ p^{y} + c \ p^{z} + t^{x} \\ d \ p^{x} + e \ p^{y} + f \ p^{z} + t^{y} \\ g \ p^{x} + h \ p^{y} + i \ p^{z} + t^{z} \end{bmatrix}$$

$$J = \begin{bmatrix} a \ \partial p'^{x} / \partial p^{y} \ \partial p'^{x} / \partial p^{y} \ \partial p'^{x} / \partial p^{z} \\ \partial p'^{y} / \partial p^{x} \ \partial p'^{y} / \partial p^{y} \ \partial p'^{y} / \partial p^{z} \\ \partial p'^{z} / \partial p^{x} \ \partial p'^{z} / \partial p^{y} \ \partial p'^{z} / \partial p^{z} \end{bmatrix}$$

-Vectors and Normals

Vectors

- Transform by Jacobian Matrix
- Matrix of partial derivatives

$$\begin{bmatrix} \vec{p}'^{x} \\ \vec{p}'^{y} \\ \vec{p}'^{z} \end{bmatrix} = \begin{bmatrix} a \ p^{x} + b \ p^{y} + c \ p^{z} + t^{x} \\ d \ p^{x} + e \ p^{y} + f \ p^{z} + t^{y} \\ g \ p^{x} + h \ p^{y} + i \ p^{z} + t^{z} \end{bmatrix}$$

$$J = \begin{bmatrix} a \ b \ \partial p'^{x} / \partial p^{z} \\ \partial p'^{y} / \partial p^{x} \ \partial p'^{y} / \partial p^{y} \ \partial p'^{y} / \partial p^{z} \\ \partial p'^{z} / \partial p^{x} \ \partial p'^{z} / \partial p^{y} \ \partial p'^{z} / \partial p^{z} \end{bmatrix}$$

-Vectors and Normals

Vectors

- Transform by Jacobian Matrix
- Matrix of partial derivatives

•
$$\begin{bmatrix} \vec{p}'^{x} \\ \vec{p}'^{y} \\ \vec{p}'^{z} \end{bmatrix} = \begin{bmatrix} a \ p^{x} + b \ p^{y} + c \ p^{z} + t^{x} \\ d \ p^{x} + e \ p^{y} + f \ p^{z} + t^{y} \\ g \ p^{x} + h \ p^{y} + i \ p^{z} + t^{z} \end{bmatrix}$$
•
$$J = \begin{bmatrix} a \ b \ c \\ \partial p'^{y} / \partial p^{x} \ \partial p'^{y} / \partial p^{y} \ \partial p'^{y} / \partial p^{z} \\ \partial p'^{z} / \partial p^{x} \ \partial p'^{z} / \partial p^{y} \ \partial p'^{z} / \partial p^{z} \end{bmatrix}$$

-Vectors and Normals

Vectors

- Transform by Jacobian Matrix
- Matrix of partial derivatives

•
$$\begin{bmatrix} \vec{p}'^{x} \\ \vec{p}'^{y} \\ \vec{p}'^{z} \end{bmatrix} = \begin{bmatrix} a \ p^{x} + b \ p^{y} + c \ p^{z} + t^{x} \\ d \ p^{x} + e \ p^{y} + f \ p^{z} + t^{y} \\ g \ p^{x} + h \ p^{y} + i \ p^{z} + t^{z} \end{bmatrix}$$

•
$$J = \begin{bmatrix} a \ b \ c \\ d \ e \ f \\ \partial p'^{z} / \partial p^{x} \ \partial p'^{z} / \partial p^{y} \ \partial p'^{z} / \partial p^{z} \end{bmatrix}$$

-Vectors and Normals

Vectors

- Transform by Jacobian Matrix
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$$\begin{bmatrix} \vec{p}'^{x} \\ \vec{p}'^{y} \\ \vec{p}'^{z} \end{bmatrix} = \begin{bmatrix} a \ p^{x} + b \ p^{y} + c \ p^{z} + t^{x} \\ d \ p^{x} + e \ p^{y} + f \ p^{z} + t^{y} \\ g \ p^{x} + h \ p^{y} + i \ p^{z} + t^{z} \end{bmatrix}$$

$$J = \begin{bmatrix} a \ b \ c \\ d \ e \ f \\ g \ h \ i \end{bmatrix}$$

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Upper-left 3x3

-Vectors and Normals

Normals

- Normal should remain perpendicular to tangent vector
- $\blacktriangleright \vec{n} \cdot \vec{v} = \vec{n}' \cdot \vec{v}' = 0$
- $\blacktriangleright \vec{n'} = \vec{n} J^{-1}$
- Multiply by inverse on right
- OR multiply column normal by inverse transpose

-Vectors and Normals

Normals

- Normal should remain perpendicular to tangent vector
- $\vec{n} \cdot \vec{v} = \vec{n}' \cdot \vec{v}' = 0$
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Vectors and Normals

Normals

Normal should remain perpendicular to tangent vector

$$\vec{n} \cdot \vec{v} = \vec{n}' \cdot \vec{v}' = 0 \begin{bmatrix} n_x & n_y & n_z \end{bmatrix} \begin{bmatrix} v^x \\ v^y \\ v^z \end{bmatrix} = 0$$

- $\blacktriangleright n' = \vec{n}J^-$
- Multiply by inverse on right
- OR multiply column normal by inverse transpose

-Vectors and Normals

Normals

Normal should remain perpendicular to tangent vector

$$\vec{n} \cdot \vec{v} = \vec{n}' \cdot \vec{v}' = 0 [n_x \quad n_y \quad n_z] I \begin{bmatrix} v^x \\ v^y \\ v^z \end{bmatrix} = 0$$

 $\blacktriangleright \vec{n'} = \vec{n} J^{-1}$

Multiply by inverse on right

OR multiply column normal by inverse transpose

-Vectors and Normals

Normals

Normal should remain perpendicular to tangent vector

$$\vec{n} \cdot \vec{v} = \vec{n}' \cdot \vec{v}' = 0 [n_x \quad n_y \quad n_z] J^{-1} J \begin{bmatrix} v^x \\ v^y \\ v^z \end{bmatrix} = 0$$

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-Vectors and Normals

Normals

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$$\vec{n} \cdot \vec{v} = \vec{n}' \cdot \vec{v}' = 0$$

$$[n_x \quad n_y \quad n_z] J^{-1} \begin{bmatrix} v'^x \\ v'^y \\ v'^z \end{bmatrix} = 0$$

 $\blacktriangleright \vec{n'} = \vec{n} J^{-1}$

Multiply by inverse on right

OR multiply column normal by inverse transpose

-Vectors and Normals

Normals

Normal should remain perpendicular to tangent vector

$$\vec{n} \cdot \vec{v} = \vec{n}' \cdot \vec{v}' = 0$$

$$\begin{bmatrix} n_x & n_y & n_z \end{bmatrix} \begin{bmatrix} v'^x \\ v'^y \\ v'^z \end{bmatrix} = 0$$

$$\vec{n}' = \vec{n} J^{-1}$$

- Multiply by inverse on right
- OR multiply column normal by inverse transpose

-Vectors and Normals

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$$\blacktriangleright \vec{n'} = \vec{n} J^{-1}$$

- Multiply by inverse on right
- OR multiply column normal by inverse transpose

-Nested Transforms

Nesting

- Room
 - Desk
 - Student
 - Book
 - Notebook
 - Desk
 - Student
 - Notebook
 - Table
 - Laptop
 - Blackboard
 - Chalk
 - Chalk
 - Eraser

-Nested Transforms

Matrix Stack

- Remember transformation, return to it later
- Push a copy, modify the copy, pop
 - RiBeginTransform()/RiEndTransform()
 - glPushMatrix()/glPopMatrix()
- Keep matrix and update matrix and inverse

Push and pop both