

PERMUTATIONS: EXAMPLES & PROBLEMS

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Consider the following permutations:

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 9 & 11 & 7 & 8 & 10 & 2 & 1 & 5 & 3 & 4 & 6 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 6 & 3 & 10 & 2 & 11 & 1 & 5 & 4 & 9 & 8 \end{pmatrix}$$

When written as a product of cycles, we have

$$a = (1, 9, 3, 7) (2, 11, 6) (4, 8, 5, 10)$$

and

$$b = (1, 7) (2, 6, 11, 8, 5) (3) (4, 10, 9) = (1, 7) (2, 6, 11, 8, 5) (4, 10, 9)$$

The inverse a^{-1} is:

$$a^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 6 & 9 & 10 & 8 & 11 & 3 & 4 & 1 & 5 & 2 \end{pmatrix}$$

$$= (7, 3, 9, 1) (6, 11, 2) (10, 5, 8, 4)$$

Moreover the product ab of a and b is:

$$ab = [(1, 9, 3, 7) (2, 11, 6) (4, 8, 5, 10)] \cdot [(1, 7) (2, 6, 11, 8, 5) (4, 10, 9)]$$

$$= (1) (2) (3, 7, 9, 8, 10) (4) (5, 11) (6) = (3, 7, 9, 8, 10) (5, 11)$$

Problem 1. $ab = ?$

Problem 2. $b^{-1} = ?$