



CMSC 461, Database Management Systems  
Spring 2018

# Chapter 6 – Formal Relational Query Languages

These slides are based on “Database System Concepts” book and slides, 6<sup>th</sup> edition, and the 2009/2012 CMSC 461 slides by Dr. Kalpakis

# Logistics

- Homework 1 due Wednesday 2/7/2018
- Dr. Sleeman out on Wednesday
  - Class will still meet, guest lecturer
- Project is posted, we will review today
- Phase 1 of project is due 2/14/2018

# Lecture Outline

- Intro to Relational Algebra
- Fundamental Operations
- Additional Operations
- Summary
- In Class Exercise

# Lecture Outline

- *Intro to Relational Algebra*
- Fundamental Operations
- Additional Operations
- Summary
- In Class Exercise

# Relational Algebra

- A procedural query language based on the mathematical theory of sets that is the foundation of commercial DBMS query languages
- The operations typically take one or two relations as inputs and give a new relation as a result
- Can build expressions using multiple relational operations

# Relational Algebra

- What is the difference between a procedural language and a non-procedural language?

# Relational Algebra

- Procedural languages tell you how to process a query (a sequence of steps provide the how)
- Non-Procedural or declarative languages tell you what to process but not how to process

# Relational Algebra

- Six basic operators
  - select:  $\sigma$
  - project:  $\pi$
  - union:  $\cup$
  - set difference:  $-$
  - Cartesian product:  $\times$
  - rename:  $\rho$

# Lecture Outline

- Intro to Relational Algebra
- **Fundamental Operations**
- Additional Operations
- Summary
- In Class Exercise

# Select Operation

$$\sigma_p(r) = \{t \mid t \in r \text{ and } p(t)\}$$

Where  $p$  is the **selection predicate**, a formula in propositional calculus consisting of **terms** connected by logical operators  $\wedge$  (**and**),  $\vee$  (**or**),  $\neg$  (**not**)

Each **term** is one of:

<attribute>  $op$  <attribute>

<attribute>  $op$  <constant>

where  $op$  is one of:  $=$   $\neq$   $>$   $\geq$   $<$   $\leq$

# Select Operation

Instructor.dept\_name = Department.dept\_name (Simple pred)

Instructor.dept\_name='Finance' (Simple pred)

Instructor.dept\_name = Department.dept\_name or Instructor.Name = 'Wu' (Boolean Combination pred)

Instructor.dept\_name = Department.dept\_name and

Instructor.Name = 'Wu' (Boolean Combination pred)

Not Instructor.Name = 'Wu' (Boolean Combination pred)

# Select Operation

A	B	C	D
$\alpha$	$\alpha$	1	7
$\alpha$	$\beta$	5	7
$\beta$	$\beta$	12	3
$\beta$	$\beta$	23	10

$$\sigma_{A=B \wedge D > 5}(r)$$

r

# Select Operation

A	B	C	D
$\alpha$	$\alpha$	1	7
$\alpha$	$\beta$	5	7
$\beta$	$\beta$	12	3
$\beta$	$\beta$	23	10

r

A	B	C	D
$\alpha$	$\alpha$	1	7
$\beta$	$\beta$	23	10

$\sigma_{A=B \wedge D > 5}(r)$

# Example Select Operation

$\sigma_{dept\_name="Physics"}(instructor)$

# Project Operation

$$\Pi_{A_1, A_2, \dots, A_k}(r)$$

Where  $A_1, A_2$  are attribute names and  $r$  is a relation name.

The result is defined as the relation of  $k$  columns obtained by dropping the columns that are not listed

Duplicate rows removed from result, since relations are sets

A	B	C
$\alpha$	10	1
$\alpha$	20	1
$\beta$	30	1
$\beta$	40	2

A	C
$\alpha$	1
$\alpha$	1
$\beta$	1
$\beta$	2

=

A	C
$\alpha$	1
$\beta$	1
$\beta$	2

$r$

$\Pi_{A,C}(r)$

# Example Project Operation

To eliminate the *dept\_name* attribute of *instructor*

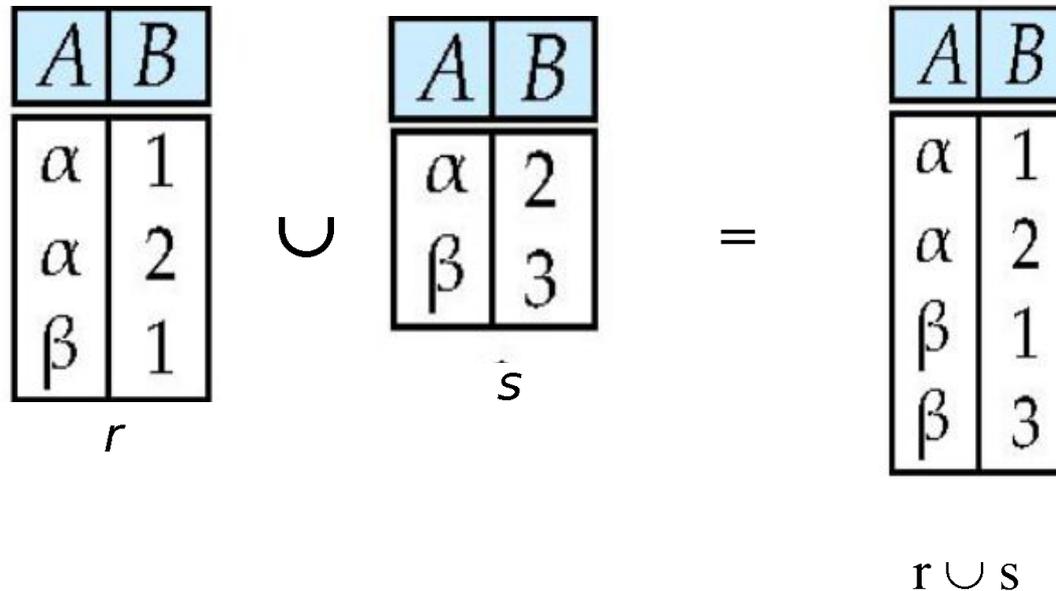
$$\Pi_{ID, name, salary} (instructor)$$

# Union Operation

$$r \cup s = \{t \mid t \in r \text{ or } t \in s\}$$

For  $r \cup s$  to be valid, these relations have to be **union compatible**.

- $r$  and  $s$  must have the same **arity** (same number of attributes)
- the domains of the corresponding attributes must be **compatible**  
(example: 2<sup>nd</sup> column of  $r$  deals with the same type of values as does the 2<sup>nd</sup> column of  $s$ )



# Example Union Operation

To find all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or in both

$$\pi_{course\_id} (\sigma_{semester="Fall" \wedge year=2009} (section)) \cup \pi_{course\_id} (\sigma_{semester="Spring" \wedge year=2010} (section))$$

# Set Difference Operation

$$r - s = \{t \mid t \in r \text{ and } t \notin s\}$$

Set difference must be taken between compatible relations.

- $r$  and  $s$  must have the same arity
- Attribute domains of  $r$  and  $s$  must be compatible

A	B
$\alpha$	1
$\alpha$	2
$\beta$	1

$r$

-

A	B
$\alpha$	2
$\beta$	3

$s$

=

A	B
$\alpha$	1
$\beta$	1

$r - s$

# Example Set Difference Operation

To find all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$\begin{aligned} & \pi_{course\_id} (\sigma_{semester="Fall" \wedge year=2009} (section)) \\ - & \pi_{course\_id} (\sigma_{semester="Spring" \wedge year=2010} (section)) \end{aligned}$$

# Cartesian-Product Operation

$$r \times s = \{t \ q \mid t \in r \text{ and } q \in s\}$$

Assume that attributes of  $r$  and  $s$  are disjoint. If attributes of  $r$  and  $s$  are not disjoint, then renaming must be used.

A	B
$\alpha$	1
$\beta$	2

$r$

x

C	D	E
$\alpha$	10	a
$\beta$	10	a
$\beta$	20	b
$\gamma$	10	b

$s$

=

A	B	C	D	E
$\alpha$	1	$\alpha$	10	a
$\alpha$	1	$\beta$	10	a
$\alpha$	1	$\beta$	20	b
$\alpha$	1	$\gamma$	10	b
$\beta$	2	$\alpha$	10	a
$\beta$	2	$\beta$	10	a
$\beta$	2	$\beta$	20	b
$\beta$	2	$\gamma$	10	b

$r \times s$

# Example Cartesian-Product Operation

To find the names of all instructors in the Physics department together with the `course_id` of all courses they taught:

$$\Pi_{name, course\_id} (\sigma_{instructor.ID=teaches.ID} (\sigma_{depart\_name = "Physics"} (instructor \times teaches)))$$

For  $r = instructor \times teaches$ :

(instructor.ID, name, dept\_name, salary  
teaches.ID, course\_id, sec\_id, semester, year)

# Composition of Operations

Can build expressions using multiple operations  
**Relational-algebra expression** – composition of relational-algebra operations

Example:  $\sigma_{A=C}(r \times s)$

A	B	C	D	E
$\alpha$	1	$\alpha$	10	a
$\alpha$	1	$\beta$	10	a
$\alpha$	1	$\beta$	20	b
$\alpha$	1	$\gamma$	10	b
$\beta$	2	$\alpha$	10	a
$\beta$	2	$\beta$	10	a
$\beta$	2	$\beta$	20	b
$\beta$	2	$\gamma$	10	b

$r \times s$

A	B	C	D	E
$\alpha$	1	$\alpha$	10	a
$\beta$	2	$\beta$	10	a
$\beta$	2	$\beta$	20	b

$\sigma_{A=C}(r \times s)$

# Rename Operation

$$\rho_x(E)$$

Returns the expression  $E$  under the name  $X$

If a relational-algebra expression  $E$  has arity  $n$ ,  
then

$$\rho_{x(A_1, A_2, \dots, A_n)}(E)$$

returns the result of expression  $E$  under the name

$X$ , and with the

attributes renamed to  $A_1, A_2, \dots, A_n$ .

# Rename Operation

- Allows us to name, and therefore to refer to, the results of relational-algebra expressions.
- Allows us to refer to a relation by more than one name.

# Example Rename Operation

$\sigma_{instructor.salary < d.salary} (instructor \times \rho_d (instructor))$

Using the rename operation to rename a reference to the instructor table so the relation can be referenced twice without ambiguity

# Example 2 Rename Operation

$\rho$

$d(\text{InstructorID}, \text{InstructorName}, \text{InstructorDepartName}, \text{InstructorSalary})$  (*instructor*)

Using the rename operation to rename attributes

# Alternative – Positional Notation

Name attributes of relation implicitly

- \$1 – first attribute, \$2 – second attribute ...

Also applies to results of relational-algebra operations

# Alternative – Positional Notation

<i>ID</i>	<i>name</i>	<i>dept_name</i>	<i>salary</i>
22222	Einstein	Physics	95000
12121	Wu	Finance	90000
32343	El Said	History	60000
45565	Katz	Comp. Sci.	75000
98345	Kim	Elec. Eng.	80000
76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
58583	Califieri	History	62000
83821	Brandt	Comp. Sci.	92000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
76543	Singh	Finance	80000

(a) The *instructor* table

**What is the output?  $\prod_{\$4} (\sigma_{\$4 < \$8} (\text{instructor } X \text{ instructor}))$**

# Example Queries

Find the largest salary in the university

- Step 1: find instructor salaries that are less than some other instructor salary (i.e. not maximum)
  - using a copy of *instructor* under a new name *d*

$$\Pi_{instructor.salary} (\sigma_{instructor.salary < d.salary} (instructor \times \rho_d (instructor)))$$

- Step 2: Find the largest salary

$$\Pi_{salary} (instructor) - \Pi_{instructor.salary} (\sigma_{instructor.salary < d.salary} (instructor \times \rho_d (instructor)))$$

# Example Queries

Find the names of all instructors in the Physics department, along with the *course\_id* of all courses they have taught

$$\Pi_{instructor.ID, course\_id} \left( \sigma_{dept\_name="Physics"} \left( \sigma_{instructor.ID=teaches.ID} (instructor \times teaches) \right) \right)$$
$$\Pi_{instructor.ID, course\_id} \left( \sigma_{instructor.ID=teaches.ID} \left( \sigma_{dept\_name="Physics"} (instructor \times teaches) \right) \right)$$

# Experimenting with Relational Algebra - Relational

<http://ltworf.github.io/relational/>

On Github <https://github.com/ltworf/relational/>

Query := Query BinaryOp Query

Query := (Query)

Query :=  $\sigma$  PYExprWithoutParenthesis (Query) |  $\sigma$  (PYExpr)  
(Query)

Query :=  $\pi$  FieldList (Query)

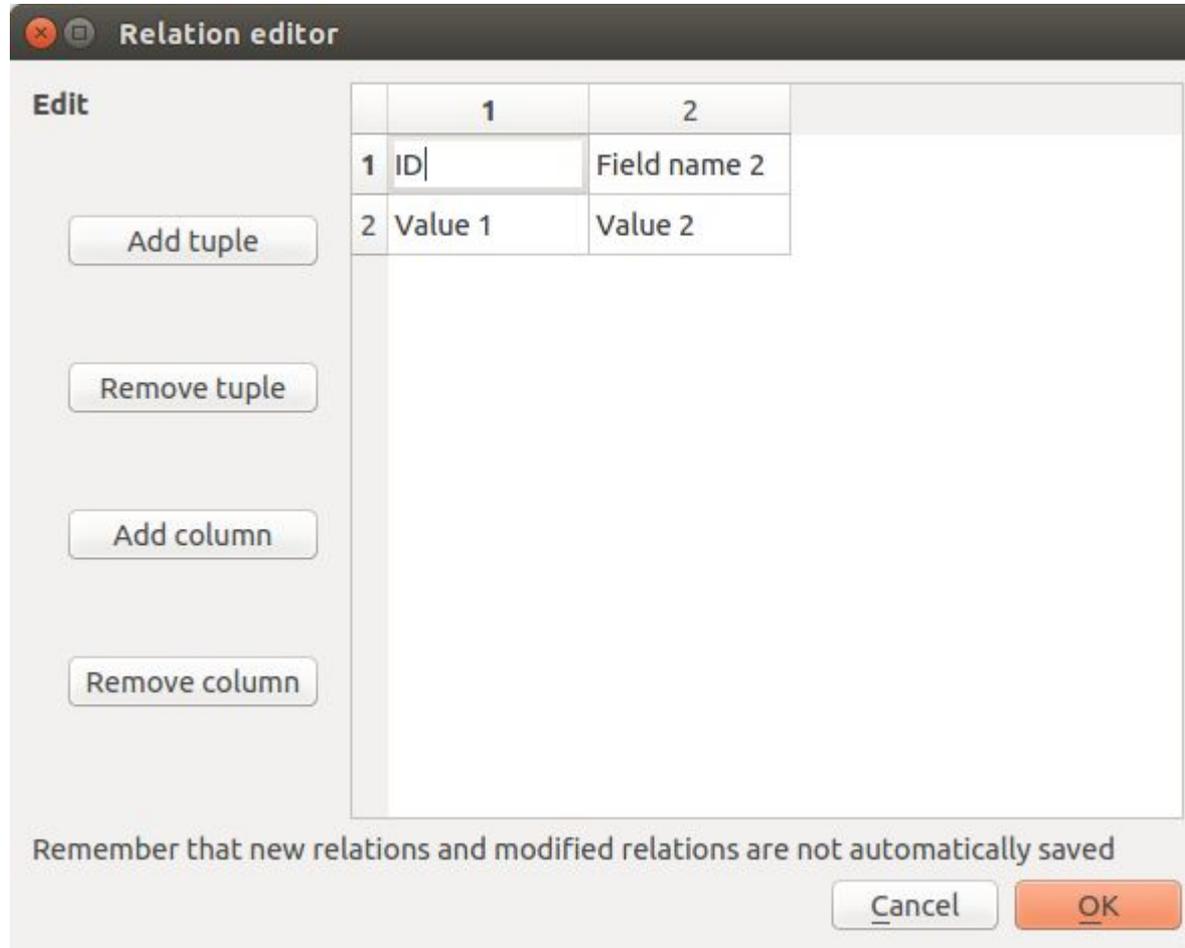
Query :=  $\rho$  RenameList (Query)

FieldList := Ident | Ident , FieldList

RenameList := Ident  $\Rightarrow$  Ident | Ident  $\Rightarrow$  Ident , RenameList

BinaryOp := \* | - |  $\square$  |  $\square$  |  $\div$  |  $\square\square$  |  $\square$ LEFT $\square$  |  $\square$ RIGHT $\square$  |  $\square$ FULL $\square$

# Relational – Creating a relation



The screenshot shows a window titled "Relation editor" with a table and several control buttons. The table has two columns labeled "1" and "2". The first row contains "ID" and "Field name 2". The second row contains "Value 1" and "Value 2".

	1	2
1	ID	Field name 2
2	Value 1	Value 2

Buttons on the left side of the window:

- Add tuple
- Remove tuple
- Add column
- Remove column

At the bottom of the window, there is a warning message: "Remember that new relations and modified relations are not automatically saved". Below this message are two buttons: "Cancel" and "OK".

# Adding tuples - Relational

Relation editor

Edit

Add tuple

Remove tuple

Add column

Remove column

	1	2	3	4
1	ID	name	depart_name	salary
2	10101	Srinivasan	Comp. Sci.	65000
3	12121	Wu	Finance	90000
4	15151	Mozart	Music	40000

Remember that new relations and modified relations are not automatically saved

Cancel OK

# Select Operation - Relational

The screenshot shows a relational database query editor interface. It is divided into several sections:

- Menu:** Contains buttons for "About" and "Survey".
- Operators:** A vertical list of buttons for various operators: \*, -, U, ∩, ÷, ▷▷, ▷LEFT◁, ▷RIGHT◁, ▷FULL◁, π, σ, ρ, and →.
- Table:** A table with columns ID, name, depart\_name, and salary. It contains one row: 15151, Mozart, Music, 40000.
- Relations:** A list of relations including \_last12, \_last3, \_last4, \_last5, \_last6, \_last7, \_last8, \_last9, Cat, instructor, and Pet. Below the list are buttons for "New relation", "Load relation", "Save relation", "Edit relation", and "Unload relation".
- Attributes:** A list of attributes including course\_id, section, semester, year, and ID.
- Query Editor:** A large text area containing a sequence of SQL operations:

```
_last6 = πname,salary(instructor)
_last7 = instructor * teaches
_last8 = σ(depart_name=='Music')(instructor * teaches)
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last7 = instructor * teaches
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last12 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last8 = σ(depart_name=='Music')(instructor * teaches)
_last7 = instructor * teaches
_last6 = πname,salary(instructor)
_last5 = σ(depart_name=='Music')(instructor)
```
- Buttons:** "Optimize", "Undo optimize", and "Clear history" are located below the query editor.
- Execution:** At the bottom, there is a text input field containing "\_last18 = σ(depart\_name=='Music')(instructor)", followed by a button with a cross icon and an "Execute" button.

# Project Operation - Relational

The interface is divided into several sections:

- Menu:** Contains buttons for "About" and "Survey".
- Operators:** A vertical stack of buttons for relational operators: \*, -, U, ∩, ÷, ⋈, ▷LEFT◁, ▷RIGHT◁, ▷FULL◁, π, σ, ρ, and →.
- Table:** A table with columns "name" and "salary".

name	salary
Srinivasan	65000
Wu	90000
Mozart	40000
- Relations:** A list of relations including \_last12, \_last3, \_last4, \_last5, \_last6, \_last7, \_last8, \_last9, Cat, instructor, and Pet. Below the list are buttons for "New relation", "Load relation", "Save relation", "Edit relation", and "Unload relation".
- Attributes:** A list of attributes including course\_id, section, semester, year, and ID.
- Query Editor:** A large text area containing a sequence of relational algebra expressions:

```
_last5 = σ(depart_name=='Music')(instructor)
_last6 = πname,salary(instructor)
_last7 = instructor * teaches
_last8 = σ(depart_name=='Music')(instructor * teaches)
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last7 = instructor * teaches
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last12 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last8 = σ(depart_name=='Music')(instructor * teaches)
_last7 = instructor * teaches
_last6 = πname,salary(instructor)
```
- Buttons:** "Optimize", "Undo optimize", and "Clear history" are located below the query editor.
- Execution:** At the bottom, there is a text input field containing "\_last17 = πname,salary(instructor)", a button with a cross icon, and an "Execute" button.

# Cartesian Product - Relational

The screenshot shows a query editor interface with several components:

- Menu:** Buttons for "About" and "Survey".
- Operators:** A vertical list of operators including  $*$ ,  $-$ ,  $\cup$ ,  $\cap$ ,  $\div$ ,  $\bowtie$ ,  $\triangleright$ LEFT $\triangleleft$ ,  $\triangleright$ RIGHT $\triangleleft$ ,  $\triangleright$ FULL $\triangleleft$ ,  $\pi$ ,  $\sigma$ ,  $\rho$ , and  $\rightarrow$ .
- Table:** A table with columns ID, name, depart\_name, and salary. It contains one row: 15151, Mozart, Music, 40000.
- Relations:** A list of relations including \_last4, \_last5, \_last6, \_last7, \_last8, \_last9, Cat, instructor, Pet, teaches, and Type. Below the list are buttons for "New relation", "Load relation", "Save relation", "Edit relation", and "Unload relation".
- Attributes:** A list of attributes including course\_id, section, semester, year, and instructor\_ID.
- Query Editor:** A text area containing a query with multiple lines of SQL. The query starts with `_last6 =  $\pi$ name,salary(instructor)` and ends with `_last5 =  $\sigma$ (depart_name=='Music')(instructor)`. The query involves a Cartesian product (`instructor * teaches`) followed by several projection and selection operations.
- Error Dialog:** A modal dialog box titled "Error" with an information icon. The message reads: "Check your query! Unable to perform product on relations with colliding attributes". An "OK" button is at the bottom right.
- Buttons:** "Optimize", "Undo optimize", and "Clear history" are located below the query editor.
- Execution:** At the bottom, there is a text input field containing `_last7 = instructor * teaches`, followed by a button with a cross icon and an "Execute" button.

# Cartesian Product - Relational

**Menu**

About

Survey

**Operators**

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ID	name	depart_name	salary	course_id	section	semester	year	instructor_ID
15151	Mozart	Music	40000	CS-101	1	Fall	2009	10101
15151	Mozart	Music	40000	CS-315	1	Spring	2010	10101
10101	Srinivasan	Comp. Sci.	65000	CS-315	1	Spring	2010	10101
12121	Wu	Finance	90000	FIN-201	1	Spring	2010	12121
10101	Srinivasan	Comp. Sci.	65000	FIN-201	1	Spring	2010	12121
15151	Mozart	Music	40000	FIN-201	1	Spring	2010	12121
12121	Wu	Finance	90000	CS-315	1	Spring	2010	10101
15151	Mozart	Music	40000	MU-199	1	Spring	2010	15151
10101	Srinivasan	Comp. Sci.	65000	CS-101	1	Fall	2009	10101
12121	Wu	Finance	90000	CS-101	1	Fall	2009	10101
12121	Wu	Finance	90000	MU-199	1	Spring	2010	15151
10101	Srinivasan	Comp. Sci.	65000	MU-199	1	Spring	2010	15151

```

_last4 = ρ Name⇒ID,Color⇒Markings(Cat)
_last5 = σ(depart_name=='Music')(instructor)
_last6 = πname,salary(instructor)
_last7 = instructor * teaches
_last8 = σ(depart_name=='Music')(instructor * teaches)
_last9 = σ(ID=instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last7 = instructor * teaches
_last9 = σ(ID=instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last12 = σ(ID=instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last9 = σ(ID=instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last8 = σ(depart_name=='Music')(instructor * teaches)
_last7 = instructor * teaches

```

Optimize    Undo optimize    Clear history

\_last16 = instructor \* teaches

**Relations**

\_last12

\_last3

\_last4

\_last5

\_last6

\_last7

\_last8

\_last9

Cat

instructor

Pet

teaches

New relation

Load relation

Save relation

Edit relation

Unload relation

**Attributes**

course\_id

section

semester

year

ID

Execute

# Relational Algebra Expressions - Relational

**Menu**

About

Survey

**Operators**

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ID	name	depart_name	salary	course_id	section	semester	year	instructor_ID
15151	Mozart	Music	40000	CS-101	1	Fall	2009	10101
15151	Mozart	Music	40000	CS-315	1	Spring	2010	10101
15151	Mozart	Music	40000	MU-199	1	Spring	2010	15151
15151	Mozart	Music	40000	FIN-201	1	Spring	2010	12121

```

_last3 = σ(Type=='Cat')(Pet)
_last4 = ρ Name→ID,Color→Markings(Cat)
_last5 = σ(depart_name=='Music')(instructor)
_last6 = πname,salary(instructor)
_last7 = instructor * teaches
_last8 = σ(depart_name=='Music')(instructor * teaches)
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last7 = instructor * teaches
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last12 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
_last8 = σ(depart_name=='Music')(instructor * teaches)

```

Optimize    Undo optimize    Clear history

\_last15 = σ(depart\_name=='Music')(instructor \* teaches)

**Relations**

\_last12

\_last3

\_last4

\_last5

\_last6

\_last7

\_last8

\_last9

Cat

instructor

Pet

New relation

Load relation

Save relation

Edit relation

Unload relation

**Attributes**

course\_id

section

semester

year

ID

Execute

# Relational Algebra Expressions - Relational

**Menu**

About

Survey

**Operators**

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▷RIGHT◁

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ID	name	depart_name	salary	course_id	section	semester	year	instructor_ID
15151	Mozart	Music	40000	MU-199	1	Spring	2010	15151

```

Type = πType(Pet)
Cat = σ(Type.startswith('Cat'))(Pet)
_last3 = σ(Type=='Cat')(Pet)
_last4 = ρ Name→ID,Color→Markings(Cat)
_last5 = σ(depart_name=='Music')(instructor)
_last6 = πname,salary(instructor)
_last7 = instructor * teaches
_last8 = σ(depart_name=='Music')(instructor * teaches)
_last9 = σ(ID==instructor_ID)(σ(depart_name=='Music')(instructor * teaches))
                    
```

Optimize    Undo optimize    Clear history

\_last10 = σ(ID==instructor\_ID)(σ(depart\_name=='Music')(instructor \* teaches))

**Relations**

\_last3

\_last4

\_last5

\_last6

\_last7

\_last8

\_last9

Cat

instructor

Pet

teaches

...

New relation

Load relation

Save relation

Edit relation

Unload relation

**Attributes**

course\_id

section

semester

year

ID

Execute

# Lecture Outline

- Intro to Relational Algebra
- Fundamental Operations
- **Additional Operations**
- Summary
- In Class Exercise

# Additional Operations

- We define additional operations that do not add any expressive power to the relational algebra, but that simplify common queries.
  - Set intersection
  - Natural join
  - Division
  - Assignment