

# Visual Analysis of Dimensionality Reduction for Exploring Bat Flight Kinematics in a Virtual Environment

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## Abstract

*We present an experiment to study two approaches for modeling complex skeletal motion of a bat in flight. Both the domain-enhancement approach and the data-fidelity approach use proper orthogonal decomposition (POD) to study animal locomotion. The motion was displayed in a fishtank virtual environment (VE) with a three-dimensional user interface (3D UI) that supports flight comparison. The UI design was evaluated by biologists and engineering scientists, who explored the linear combinations of individual components described by POD and subsequently uncovered and verified patterns in animal locomotion using the two approaches.*

*Our experimental results suggest that displaying experimental kinematics data in an interactive 3D visualization system gave the scientists an intuitive biological interpretation of biomechanical patterns of animal flight. In particular, feedback from experts who compared this environment with traditional two-dimensional (2D) graphs stressed the advantage of seeing inherently 3D data in a VE, letting them concentrate more readily on particular aspects of data analyses. Subsequently, they were able to notice new motion patterns as well as differences and similarities in flight behaviors. Comparison of the two modeling methods shows similarities in the types of motions presented.*

Categories and Subject Descriptors (according to ACM CCS): H.5.2 [Information interfaces and presentation]: User Interfaces—Graphical User Interface, I.6.3 [Simulation and modeling]: Application, I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Virtual Reality

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## 1. Introduction

Effective data modeling and three-dimensional user interface (3D UI) can provide much-needed help to scientists who study natural phenomena by facilitating, generation, evaluation, and refinement of scientific hypotheses and insights [vDFL\*00, SND05, FCL09]. We describe two modeling approaches for studying animal kinematics and the design and evaluation of a comparative visualization in a fish-tank virtual environment (VE) [WAB93]. We also report an experiment which helped us obtain insights about process of hypothesis generation by the biologists and engineering scientists.

The development of this work was driven by the long-term collaboration between computer scientists and biologists at

Brown University, interested in the use of modeling, simulation, and visualization to solve practical scientific problems. The biologists in the flight research group have needed a better way to compare flight and search for features in the behavior of flying animals. Accurate modeling of flying motion is difficult because the bat wing morphs its shape and has a high degree of spatial and temporal complexity.

We adopted an iterative design process and focused on early user involvement to gather task requirements and designed several prototypes. The workflow has been that computer scientists use VEs to show biological data, biologists view the visualization and generate new hypotheses, and finally computer scientists redesign modeling and visualization tool for biologists to test the new hypotheses. This work-

flow that moves from data to visualization to insight is made possible by modeling and effective interface design. This paper reports how different data representations affect biologists' ability to generate and explore new hypotheses using our VE system. This paper contributes to the computational methods that can be applied in other gait analyses, such as for humans and birds. This paper is also a first step in designing comparative visualization user interfaces; the evaluation method can be used to study other VE applications for scientific visualizations.

## 2. Related Work

VEs can provide easy-to-understand presentation of data, thus relieving users from retaining data details in their minds and directly engaging the powerful human perceptual and motor mechanisms.

One primary focus of VE design for scientific uses is data modeling. Because visualizations are data-intensive and many applications are highly complex in space and time, it is important to present the data in terms of reduced complexity so that users can discover dominant features. One reduction approach is called proper orthogonal decomposition (POD) [LLL\*02], aims to use low-dimensional descriptions to capture many of the phenomena of interest in high dimensions while retaining the dominant patterns. POD is employed in variety of disciplines, including fluid dynamics, image processing, and flight analyses [BHL93,RWI\*08,SK87,JAG\*05], but has not been used until now for bat kinematics analyses.

Another primary focus of the UI design is to facilitate visual analyses of datasets through user interaction, which can be advantageous to ease large dataset exploration. Several methods have been proposed, such as using mini-views [AM07] in a graph or a spreadsheet, so as to present task-related information all at once [SEJL04]. This approach also reduces the amount of information users have to remember, therefore minimizing cognitive load [Swe88, TM04] and enhancing users understanding of their data.

Numerous studies have quantified the efficacy of a visualization system. Notably, Saraiya made use of an insight-based method to compare several information visualization applications [SND05]. Moberts compared several feature extraction algorithms for visualizing tensor data [MVvW05], and Laidlaw and Forsberg compared fluid flow visualization methods by direct comparison of their utility to real world flow tasks [LKJ\*05,FCL09]. There are also well-known iterative VE evaluation methods suitable to different design stages [HSG\*99, SMR\*03]. While effective evaluation approach has been initiated, our work builds on top of existing approaches and combines them to study a broad range of design issues.

Other related work also includes animation methods to extract motion from videos [FRD\*2003]. Our approach to vi-



**Figure 1:** Frame from the video recorded by one of the three high-speed digital cameras. The 3D coordinates of markers attached to the bat wings are reconstructed using Direct Linear Transformation.

ualize time-varying motion is by direct display of data sequences.

## 3. Methods

We first describe how the 3D data were acquired and then discuss two approaches to reconstructing the wing trajectory, driven either by needs of visualization and the anatomical features or by fidelity to the captured data. Comparing these two approaches is important because it is not clear which of the two would be most useful to biologists.

### 3.1. Experimental Data Acquisition

Experimental bat wing kinematics data were acquired by flying a bat (*Cynopterus brachyotis*) in a wind tunnel at different speeds [SBI-A04]. Three high-speed ( $f=1000\text{Hz}$ ) digital cameras tracked the positions of 56 painted markers attached to bone joints, wing, and body (Fig. 1). Only the 17 markers attached to the joints and body were used in this study. The image data were extracted and converted into a set of 3D marker coordinates using direct linear transformation (DLT) [Hed08].

Each marker has a 3D coordinate at each point in time, therefore the bat geometry for the 17 markers at each time frame is a 51-dimensional matrix. The deformation of the bat wing during the flight is a 51-dimensional vector in each time frame. The mesh of the wing was partitioned into eighteen triangles spanning those markers distributed across distinct anatomical regions.

The data were noisy and error prone, because the bat morphed its wing dramatically in flight, causing many occluded points. The data uncertainties also came from missing

frames due to the occlusion or to impossible 3D reconstruction from 2D images.

### 3.2. Data Reconstruction

#### 3.2.1. Domain-enhancement approach

The goal of the domain-enhancement approach (Fig. 2) is to produce a smooth cyclic motion for the visualization and maintain kinematics constraints (e.g., bone length). The experimental data were first smoothed and gap-filled from the 3D reconstructed data using a high-order global polynomial fit (original global polynomial fit). The residual data were smoothed with a Fourier analysis and then the gaps were filled using an over-constrained third-order polynomial. Finally, the original global polynomial fit was added back into the data to reconstruct the final smoothed data.

As the time-varying data were a discrete set of position vectors, simply selecting a range of frames resulted in a clearly visible jump in the data. To eliminate this jump and better represent a wingbeat cycle, we applied a polynomial fit to the frames around the jump, effectively stitching over the gap to produce a smooth cyclic motion. POD was also applied to constraint the bone length between frames.

One wingbeat cycle was analyzed using POD. We represent the time evolution of POD mode coefficients as a Fourier series to construct a continuous representation of periodic bat wing motion during flight and eliminate high frequency components in the data. By discarding high-order terms in the Fourier expansions, we can eliminate high frequency components in the evolution of POD mode coefficients. Keeping a large number of Fourier and POD modes makes possible an accurate representation of the original data. Changing the number of terms in POD and Fourier expansions lets us control the amount of variance and degree of smoothness. The approximation can be used to estimate the position of the wing at any time during the cycle, including the original set of times at which the experimental data were collected.

#### 3.2.2. Data-fidelity approach

The goal of this approach was to use as much of the captured data as possible, and to avoid introducing any possible artifacts caused by data processing. The preprocessing strategy was to use an over-constrained third-order polynomial to fill in the regular gaps when missing data occur (caused by camera occlusion or impossible digitization). Where intervals of missing data did not have a sufficient number of endpoints to over-constrain the polynomial fit, a higher order polynomial was applied to the interval in which enough endpoints were available, and included all intermediate data points.

The noise present in the data was smoothed using an order 8 low-pass digital Butterworth filter [But30] with cut-off frequency 100 Hz, corresponding to a normalized value

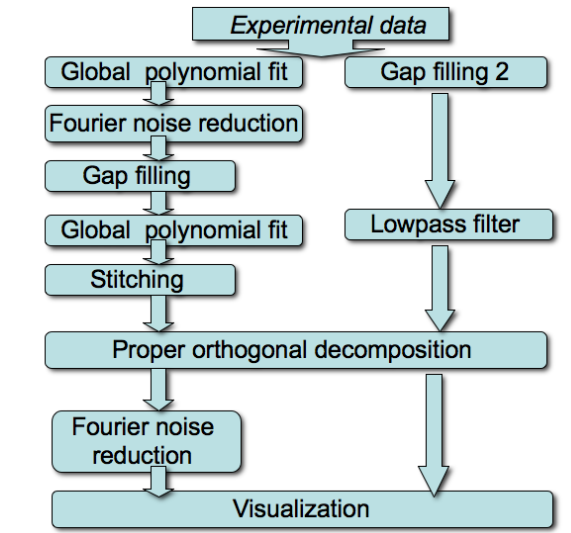


Figure 2: The data organization pipeline. The left side of the flowchart corresponds to the domain-enhancement approach, and the right side to the data-fidelity approach.

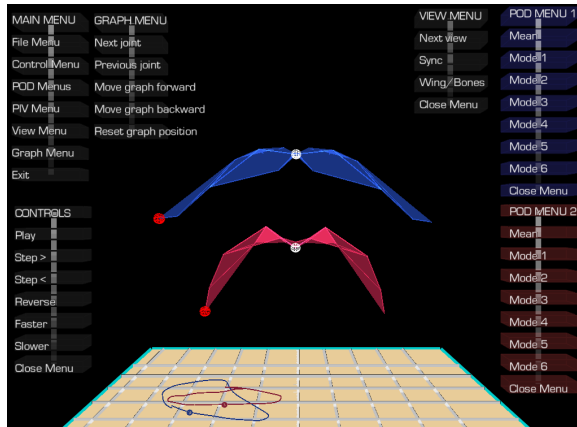
$W_n = 100/500 = 0.2$ . The frequency is about 9 times higher than the bat wingbeat frequency.

### 3.3. Proper Orthogonal Decomposition of Wing Kinematics

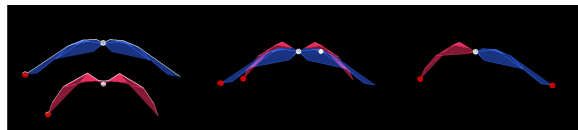
POD aims to use low-dimensional descriptions to capture many of the phenomena of interest in high dimensions [LLL\*02]. In our case, the 17 markers were placed at regions at the joints and on the body. The 3D positions of the markers in one wingbeat cycle over time (interpolated to 1000 frames) were put into a matrix in which each column describes the time-dependent displacement history of a marker in one dimension and each row described the concurrent three-dimensional marker positions at some instant in time during one wingbeat cycle. We then looked for a representation of this motion in the form of orthogonal bases using POD. The eigenvectors of the matrix form orthogonal bases. A lower-dimensional representation of the original data is constructed using the first several basis vectors (proper orthogonal modes). The percentage of the motion described by each mode was calculated based on its eigenvalue. The sum of the motion described by all bases was one. We were able to run POD on any trials that was formatted correctly.

### 3.4. User Interface

The goal of the user interface design was to enhance the scientists' ability to perform explorative tasks involving global, regional, and local examination of the datasets. Two major



**Figure 3:** Screenshot of the visualization environment in a desktop setting. Menus used to control POD component activation are color-coded to match the corresponding bat model.



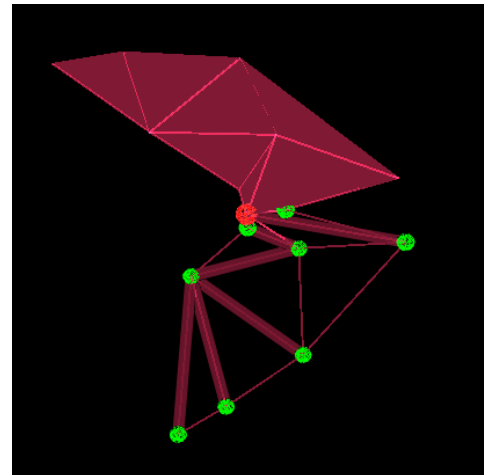
**Figure 4:** Different options for model comparison. From left to right, they are: free positioning, sternum collocation, and side-by-side collocation.

implementations were studied: interactive exploration of the POD modes and a comparative interaction method.

Since most of the tasks for which the system is used involve comparisons between two datasets, the default view presents two bat dataset models, color-coded with the associated menus to establish an intuitive visual connection between them (Fig. 3). Data that the models represent can be loaded on the fly to facilitate pairwise comparisons between flights. Each dataset represents one wingbeat, synchronized to start at the beginning of the upward recovery stroke. Models can be translated and rotated in the virtual space by dragging the sternum point with the mouse tracker.

An important feature requested by the biologists, who had used the previous version of the interface [KPB\*06], was integration of a familiar analytical tool – a scaled graph displaying a 2D projection of a marker trajectory. When users select any marker to be traced throughout the wingbeat cycle, a projection of its position, as well as its entire trajectory, is displayed on a grid plane. Having this information available in the environment reduces the amount of information that must be remembered.

We provided three options for dataset comparison (Fig. 4). In free positioning mode, the bat models could be moved



**Figure 5:** One of visual representations of marker data, a hybrid combination of bone and wing representation. Other representations include wing membrane and skeletal structure.

around the space independently or positioned side by side. In sternum collocation mode, the sternum markers of two bats were locked together to provide an intuitive overlay comparison of their motion. In side-by-side collocation mode, two datasets were “glued together” along the body line of the bat, thus making half of each dataset visible as part of the combination.

Several visual representations of the marker motion data were provided (Fig. 5). One option involved displaying the entire wing as a semi-transparent mesh. Another displayed only the bones, i.e., the segments that were part of the skeleton. In hybrid mode, one half of the model was displayed as a wing, and the other as bones; additional thin lines represented the wing membrane surface between the bones.

The user interface consists of a keyboard and mouse in a fishtank environment. Visualization control and parameter adjustment can be done using the on-screen menu system or keyboard shortcuts. The floating 3D menus are large enough for easy selection, but small enough to stay in peripheral vision. To reduce clutter, menus can be moved around the screen, opened, and closed as needed. The system provides access to playback, time-stepping, and speed controls, which are essential for effective and thorough control of the animation. Using the menus, the user can also interactively explore the contributions of individual POD components, and different combinations of components. For each model, the user can specify the modes to be included in the low-dimensional representation. A way to switch predefined viewpoints is also provided along with some default presets, including basic zoological referred views (lateral, dorsal, and ventral) and some custom views defined by our experts.

### 3.5. User Evaluation

The main goal of this study was to explore whether expert users, biologists and engineers, could use our system to gain a better understanding of bat flight. We were interested in user evaluation of the possible biological meaning of POD decomposition, and in the efficiency of the UI in performing their tasks. Our secondary goal was to gather feedback about design in order to improve the interface. A qualitative study was conducted.

**Background:** Well-known VE evaluation methods include summative and formative evaluations [HSG\*99, SMR\*03]. The importance of running these studies has been uncontested. However, Saraiya et al. argued that it is also important to evaluate the types of insight users glean from using a visualization [SND05]. They defined insight in this context to be an individual observation about the data, or a unit of discovery. In their evaluation of bioinformatics tools, they recorded “domain values” to indicate the importance of the participants’ insight. They authors found different visualization tools affect insight discovery.

Our work evaluates the data processing methods and UI design. The data processing evaluation part is similar to Saraiya’s work regarding insight discovery [SND05], and the UI evaluation is similar to Hix’s techniques [HSG\*99], who presented system design processes for constructing effective VE applications.

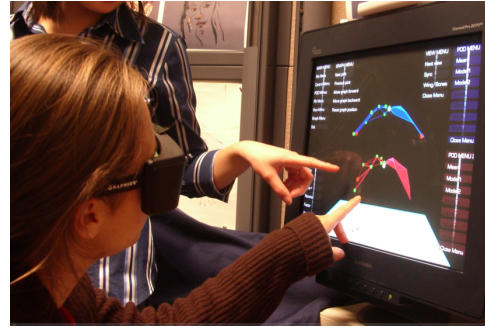
**Participants:** Eight participants (seven male, one female) participated in this study. Seven of them were researchers working with the bat flight research group (two engineers and five biologists), and one had some morphology analysis experience but no prior knowledge of the data or decomposition results. Three participants had prior user interface experience on a system prototype.

**Design:** Our study used a stereoscopic fishtank VE. Four users participated in pairs, and four others performed the tasks individually.

**Data:** Because the users were interested in exploring possible relationships between speed and flight patterns, the starting datasets corresponded to one fast and one slow flight of the same bat through a windtunnel.

**Tasks:** Participants did not have specific tasks but were asked to compare: (1) conventional methods with interactive 3D visualization, (2) full motion with mode combinations for a single run, and (3) different runs with one another. They were also asked (4) to interpret mode functionality.

**Procedure:** Participants performed the same tasks across all conditions. They received instruction about the system, the datasets, the POD method, and the study procedure, and were trained to use the fishtank VE interface and the menu system. The initial conditions for all participants included exploring the visualizations based on the two data reconstruction methods (described in Section 4.4). Users were en-



**Figure 6:** Two users comparing POD modes in a fishtank virtual reality environment. The bat datasets correspond to two different flight speeds.

couraged to explore different combinations of datasets, visualization, and analytical scenarios as they felt was useful. A think-aloud protocol [ES93] was enforced to ensure that the participants were communicating what they were thinking, doing, looking at, and discovering as they interacted with the data and the controls. The study lasted about 1.5 hours.

## 4. Results and Discussion

We concentrated on understanding how the participants were generating, evaluating, and refining their hypotheses about the data. Here we discuss the participants’ insight exploration process, and provide examples from the study that show how visualization techniques helped facilitate the process.

As soon as participants were acquainted with the system, they started exploring the data and trying to verify their general and specific knowledge about flight. In all test cases, participants also noticed spatio-temporal patterns or shape changes they wanted to explore, and this often led them to use the visualization to formulate a hypothesis they scrutinized using other features of the system.

### 4.1. View Selection

Most of the participants noticed that locking the sternum points together (thus overlapping the two models) clearly showed the detailed differences in wing motion while helping maintain a visual connection between the two patterns. Because they wanted to classify these differences, the behavior of all participants was very similar: they first cycled through the predefined viewpoints or rotated the models by hand to identify large-scale differences, and then either began tracing different markers right in the 2D projection, or cycled through the visualization representation options to find which one answered the questions best. The results showed that combining the two “halves” of the models into a side-by-side double-colored mesh (as described above) was

perceived as useful for pointing out the general idea of differences, while overlapping the models one on top of the other and switching among bone, membrane, and hybrid representations was more effective for comparing finer.

We reiterate that 2D projections of a selected marker trajectory were used by participants extensively during most tasks, and participant feedback showed that using these projections at the same time, as viewing animated motion of bat models in 3D stereo, helped the process immensely.

#### **4.2. Morphology Analysis**

Participants with morphology analysis experience concentrated mainly on the relationships among flight speed, wing shape, and joint movement in the wingbeat cycle. One participant noticed that the general wing shape before the recovery stroke (i.e. the upward motion of the wing) was different for two flight speeds and suggested that a possible reason for this was the different energetic requirements the animal had to fulfill to perform adequately in the wind conditions. To verify the initial observation, the participant used the 2D projection to trace the wingtip markers and confirmed the significance in coordinate difference between the test runs. As a result, the participant suggested analyzing quantitative energy transfer to see whether this hypothesis would indeed hold.

Like the rest of the participants, the morphologists noted the difference in foot movement patterns at different flight speeds. They also discovered that in all trials the synchronization of motion between the feet and the arms was very pronounced in the visualization – a significant observation from an evolutionary biology point of view. Seeing an almost exact similarity of motion in different wingbeats of the same test run verified their expectations. It is important to note that these relationships had not been observed earlier from the traditional 2D graphs.

#### **4.3. Aerodynamics Ideas**

As mentioned earlier, understanding complex animal flight is also an interesting problem in aerodynamics engineering besides zoology and evolutionary biology. Several participants used the visualization to explore some ideas of aerodynamic relevance. They reported that seeing the results of POD processing in a 3D environment led them to break down a very complex motion into a set of simple ones that could be intuitively analyzed from a familiar aerodynamics point of view. For example, seeing that faster flight exhibited strong vertical flapping motion of the wing in the first POD mode gave them a good sense of thrust generation in this type of flight. Likewise, overlaying two models that corresponded to two different flight speeds suggested to them that the wing angle of attack differed in these two cases; a hypothesis was that in slower flight the angle of attack had to be higher. This would increase the surface area traversed by

the flow and increase lift necessary to support the bat under slow flight conditions. This hypothesis was checked using different test case combinations for several speeds using the same visualization technique.

#### **4.4. Evaluation of Data Reconstruction Approaches**

To evaluate the two data reconstruction approaches described in Sections 4.4, the main features and differences of the two methods were first explained to the experts. After exploring the datasets using the data-fidelity approach, they were then asked to use the domain-enhancement approach and try to spot any significant differences in the resulting motion and decomposition modes. We then evaluated how the methods might have influenced the experts' insight in constructive or misleading ways.

Six participants concluded that the first approach, based on cyclic wingbeat cycle stitching and Fourier noise smoothing, was more visually fluid and felt more natural than the other. However, they found that the two approaches are also similar in representing the motion patterns and did not lead to different insights.

Two methods for data reconstruction were also compared by the POD modes. Results suggested that the data reconstruction had a large impact on the number of POD modes required to represent the motion and the amount of motion represented by each mode. With the domain-enhancement approach, the first three modes represent 82%, 10.4%, and 3.1% of the overall motion, respectively; with the data complexity approach, they represent 31.4%, 13.2% and 12% of the motion. The former approach required the first three modes, while the latter required 17 in order to recover 95% of the overall motion. Interestingly, however, both methods resulted in similar types of physical motion, at least for the first three modes.

#### **4.5. Consistency of POD Modes**

When exploring the contributions of POD modes in the context of full motion, all participants concluded that the first three modes, combined with the mean, showed most of the motion in the model, with the first mode representing the dominant flapping motion, the second mode representing the extension of the membrane with the wrist, and the third mode most probably combining most of the remaining variances in the motion of the distal digits of the hand. After this, participants wanted to see whether these relationships held under different flight conditions, so they used the file menu system to load a dataset corresponding to a different wind tunnel speed. The relationship did hold, so one of the conclusions was that the patterns of dominant motion could be described similarly across a range of flight speeds for given animal.

Even though the participants had already worked with the

data, including the POD decomposition results, using traditional analytical methods and plots, these patterns had not been clear to them before. Thus while graphs of marker positions and POD mode contributions by themselves could be useful for determining mathematical relationships, they failed to convey how the motions actually looked, and the 3D visualization helped solve this issue.

#### 4.6. The Effects of Variance Standardization on POD Modes

As mentioned above, marker traces in 2D projection described visually similar motion patterns (flapping in the first mode, membrane stretching in the second for both reconstruction approaches). However, closer examination of the marker position projections clearly showed that some small-scale specifics, such as the direction of wingtip movement, differed. When variance standardization was disabled in the second approach, the motion of the modes became very similar. This gives evidence that normalization of mean-subtracted data by its standard deviation could affect the results obtained by using the visualization.

A practical suggestion from these results might be to use the data-fidelity approach if the visualization designers want to obtain the results quickly, and to adopt the domain-enhancement approach if the goal is to take into account domain knowledge, such as preserving the bone deformation. Since so much is yet to be discovered about bat flight, the merits of these two methods need further study.

#### 4.7. Cross-Device Comparative Interaction

Biologists' comments on the comparative visualization were positive. They especially liked the 2D projection of motion traces. However, they pointed out the importance of direct visualization using numbers so as to conduct a more detailed analysis.

The biologists also found the menu system easy to learn and use. Although VE systems are often designed for a specific input device, we intended the display and control system to be used in both the fishtank and desktop VE environments, so that they could be used in both a VE lab or a biology office. As a result, we used floating menus for control to maintain consistency across different platforms. A similar interface of the three modes (overlapping, side-by-side, and random in space) was also examined in the CAVE in our lab.

#### 5. Conclusion

We have shown that seeing virtual three-dimensional representations of 3D data gave users intuitive and clear evidence of data processing approaches, flight traits and features, and guided them towards new ideas. Integrated view of data (for example, two-dimensional projection of motion traces) proved to be an effective strategy; the users had the

data always available and were able to fully focus on exploration tasks, quantifying and comparing features when necessary.

Interacting with the 3D visualization of lower-dimensional motion components also helped the users better understand the significance and functionality of those components. As a result, the experts who participated in the study mentioned some interesting potential implications for motion analysis and modeling. Characterizing the POD modes brings us a step closer to building computational and physical bat models that will effectively encompass enough of the motion to be realistic in terms of biomechanics and aerodynamics.

Although this work is a significant first step, scientific visualization for exploring POD data is still limited by the system's failure to support explicit data presentation required for scientific purposes. One solution is to integrate the spatial data presentation and information visualization methods, making possible data presentation implicitly (visually) and explicitly (numerically). A taxonomy of interactive tasks (e.g., comparison) for scientific visualization would be extremely useful in this connection. In addition, much work remains to be done in studying and formalizing the design process and in comparing different feature extraction methods, input methods, and interaction techniques for scientific visualization in VE applications.

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