Mixed Integer Program - draft3

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August 30, 2003

1 Problem Framework

Given an undirected graph with a weight and a value function defined on nodes such that both the functions map nodes to real-valued positive numbers.
Weight $w : n \rightarrow \Re$ and Value $v : n \rightarrow \Re$.
Two nodes are said to be independent of each other, if they do not have an edge between them.

2 Problem Defination

Given a undirected graph, find a set S of atmost K mutually independent nodes such that the sum of their values is maximized, subject to the constraint that the sum of their weights do not exceed a predefined weight constant W.
Input Graph $G = (V, E)$.
Let e_{ij} denote an edge from node i to node j.

$$e_{ij} = \begin{cases} 1 & \text{if } e_{ij} \in E \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Let x denote the indicator variable for node i such that

$$x_i = \begin{cases} 1 & \text{if node } i \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Formally we can define the problem as,

Maximise $\sum_{i=1}^{n} v_i x_i$,
subject to

$$\sum_{i=1}^{n} w_i x_i \leq W, \tag{3}$$

$$x_i + x_j \leq 2 - e_{ij}, and \tag{4}$$

$$x_i \in \{0, 1\} \tag{5}$$

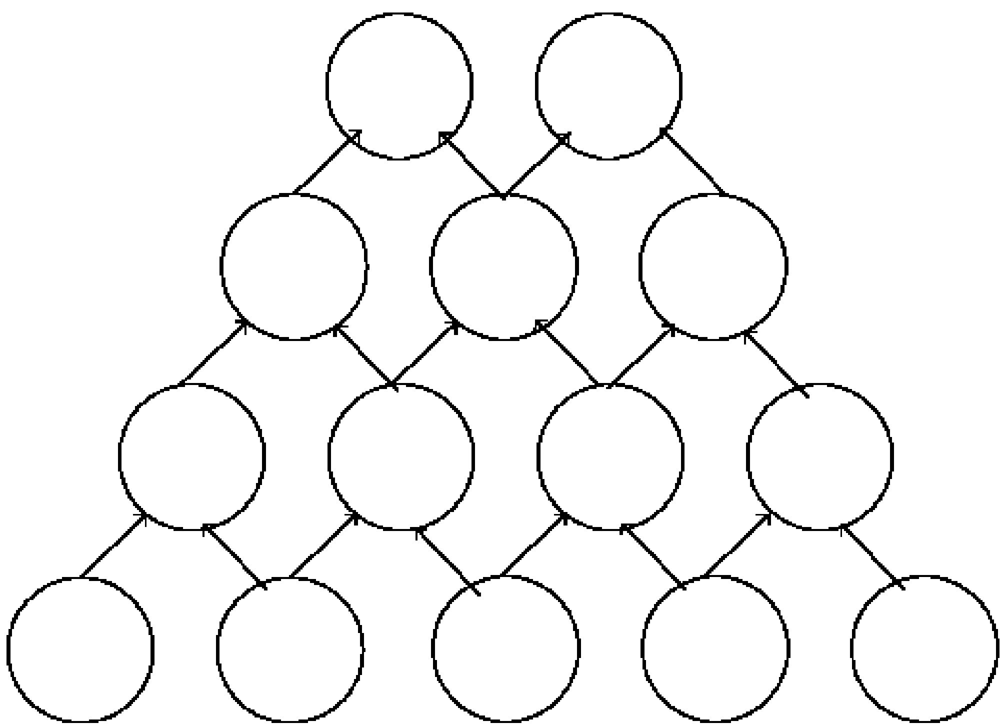
The problem can be formlated as a linear programming problem

3 Analysis

For n nodes, the number of equations formed is $O(n^2)$.
The problem is an linear interger programming problem with $O(n^2)$ equations. If the constraint for indicator variable x is relaxed so that it can take all real values from 0 to 1, we have a mixed integer program formulated as follows:

4 Using Dynamic Programming

the technique involved in the generation of the graph always renders its structure as a Pascal's triangle with nodes pointing to their adjacent nodes in one direction. It is always an acyclic directed graph, which each node having at max two incoming and two outgoing edges. To take advantge of this structure, we



can cast this problem as a dynamic programming problem

5 Characterization of the structure of an optimal solution

data structures :
Given a graph $G = (V, E)$. each node N stores the following data.
Reachable(N): all nodes reachable from node N.
Base(N): a list of leaf nodes which can reach node N.
value(N): Value of node N, a positive real number.
weight(N): Weight of node N, a positive real number.

Let n be the total number of nodes in the graph.
Let K be the maximum number of nodes allowed.
Let W be the maximum admissible weight.

we have $V = \{N_1, N_2, \ldots, N_n\}$

6 Recursive Defination of an optimal solution

let $f(K, W, V)$ be the maximum value obtained by selecting K nodes from a set of V nodes, respecting the constraint that the sum of their weights do not exceed W.
f can be recursively defined as :

$$f(K, W, V) = \max[\text{ value}(N) + f(K-1, W\text{-weight}(N), V - \text{Reachable}(\text{Base}(N)))]$$