



CMU SCS

# Large Graph Mining

*Christos Faloutsos*

CMU



# Thank you!

- Hillol Kargupta





# Outline

- **Problem definition / Motivation**
- Static & dynamic laws; generators
- Tools: CenterPiece graphs; fraud detection
- Conclusions



# Motivation

Data mining: ~ find patterns (rules, outliers)

- Problem#1: How do real graphs look like?
- Problem#2: How do they evolve?
- Problem#3: How to generate realistic graphs

## TOOLS

- Problem#4: Who is the ‘master-mind’?
- Problem#5: Fraud detection



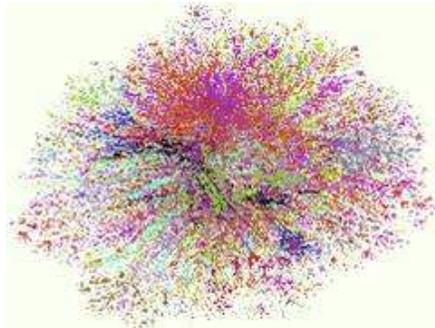
# Problem#1: Joint work with

Dr. Deepayan Chakrabarti  
(CMU/Yahoo R.L.)

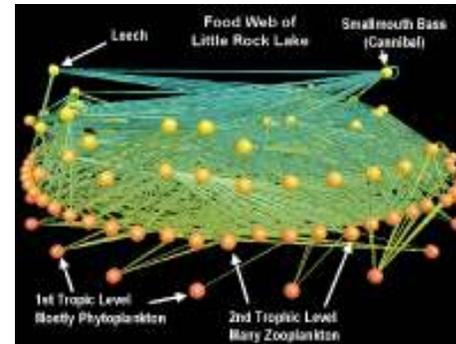




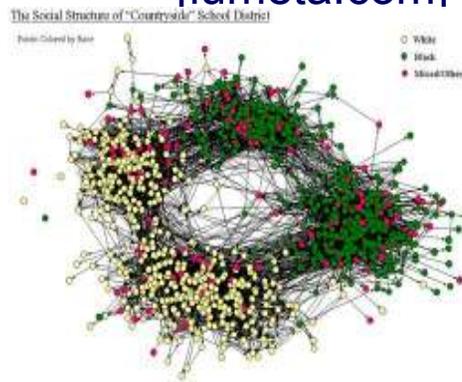
# Graphs - why should we care?



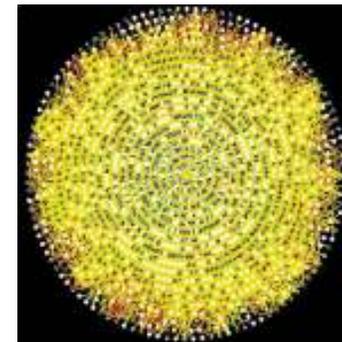
Internet Map  
[lumeta.com]



Food Web  
[Martinez '91]



Friendship Network  
[Moody '01]

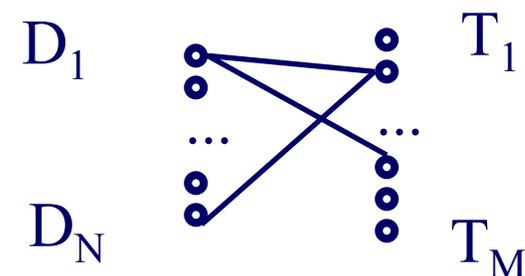


Protein Interactions  
[genomebiology.com]



# Graphs - why should we care?

- IR: bi-partite graphs (doc-terms)



- web: hyper-text graph

- ... and more:

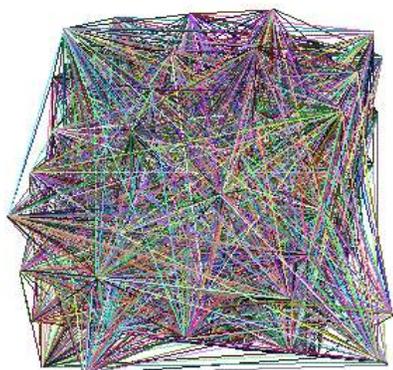


# Graphs - why should we care?

- network of companies & board-of-directors members
- ‘viral’ marketing
- web-log (‘blog’) news propagation
- computer network security: email/IP traffic and anomaly detection
- ....



# Problem #1 - network and graph mining



- How does the Internet look like?
- How does the web look like?
- What is ‘normal’/‘abnormal’?
- which patterns/laws hold?



# Graph mining

- Are real graphs random?



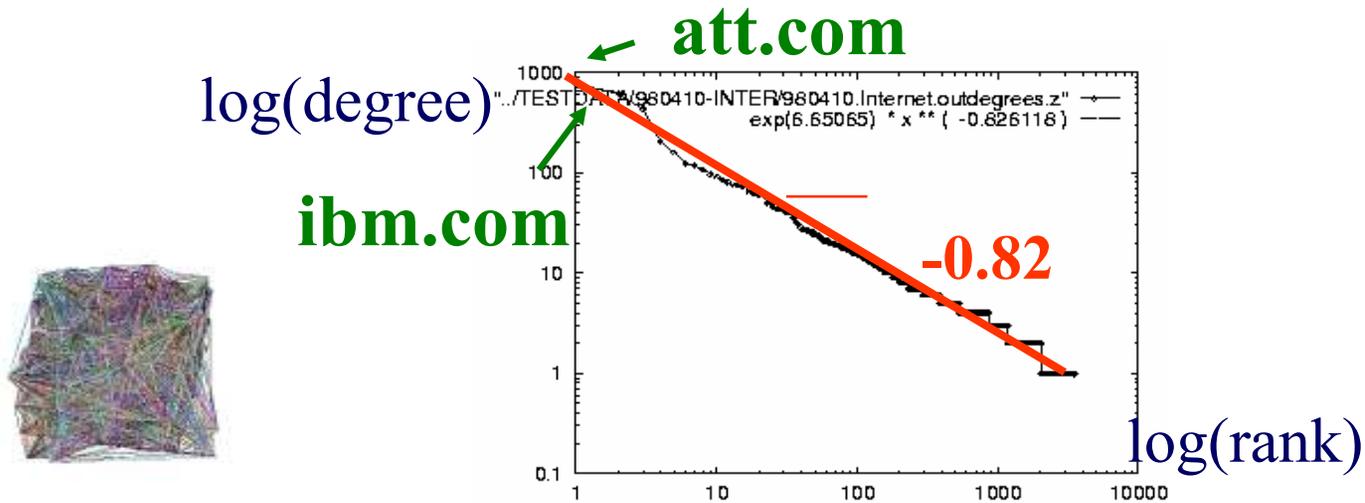
# Laws and patterns

- Are real graphs random?
- A: NO!!
  - Diameter
  - in- and out- degree distributions
  - other (surprising) patterns

# Solution#1

- Power law in the degree distribution [SIGCOMM99]

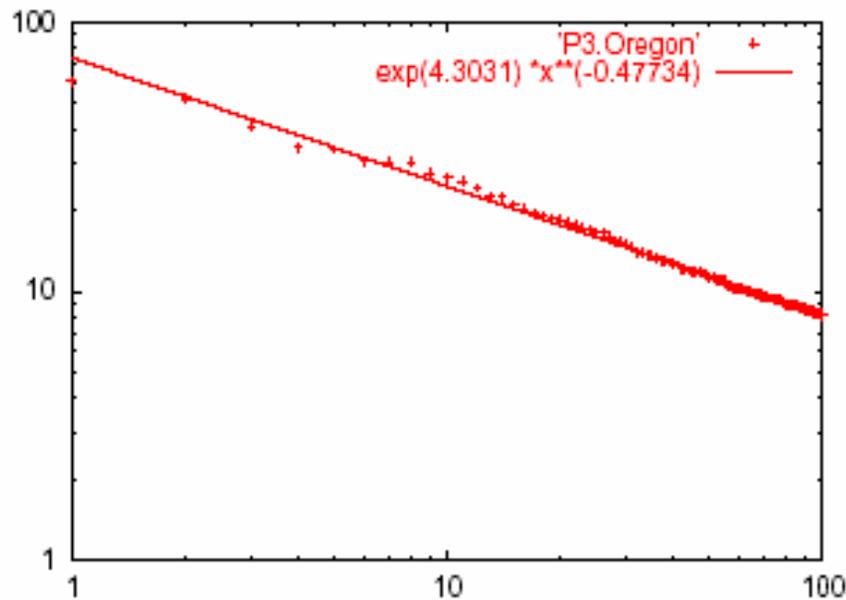
internet domains





# Solution#1': Eigen Exponent $E$

Eigenvalue



Exponent = slope

$$E = -0.48$$

May 2001

Rank of decreasing eigenvalue

- A2: power law in the eigenvalues of the adjacency matrix



**But:**

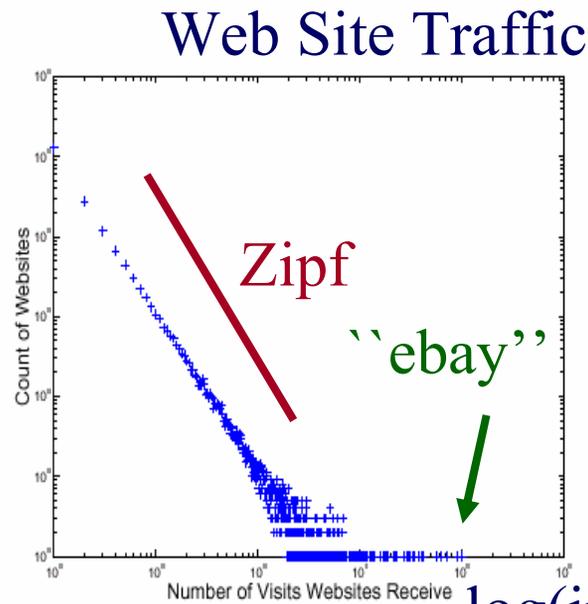
How about graphs from other domains?



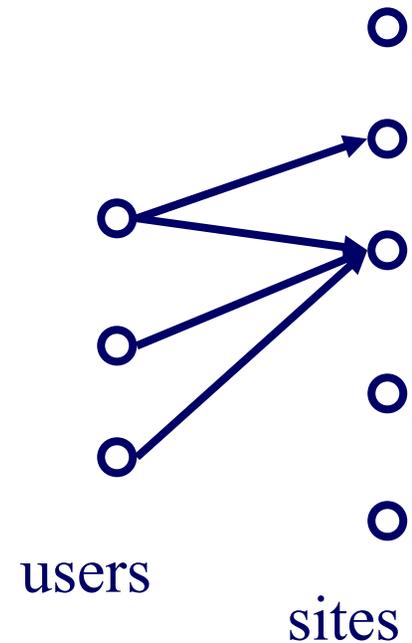
# More power laws:

- web hit counts [w/ A. Montgomery]

log(count)



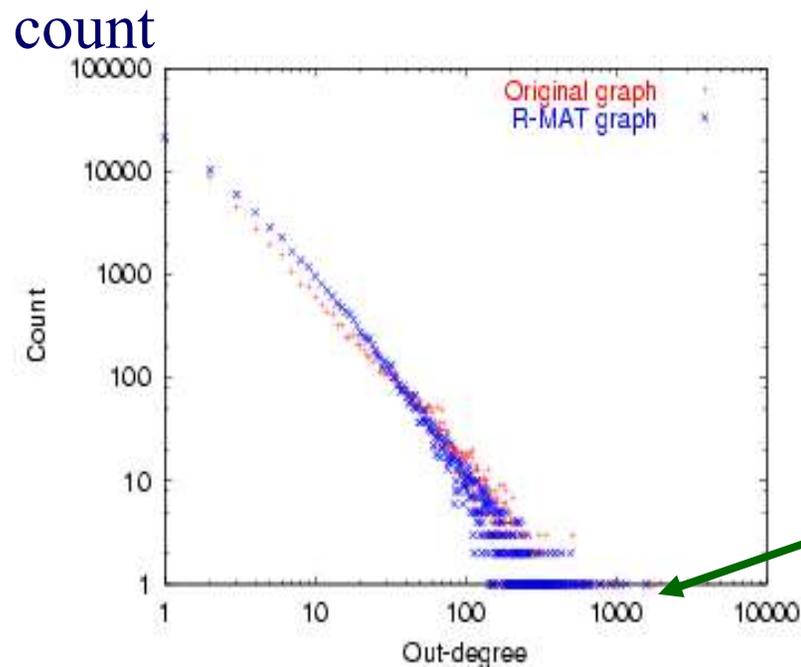
log(in-degree)





# epinions.com

- who-trusts-whom  
[Richardson + Domingos, KDD 2001]



trusts-2000-people user

(out) degree



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## TOOLS

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## Problem#2: Time evolution

- with Jure Leskovec  
(CMU/MLD)
- and Jon Kleinberg (Cornell –  
sabb. @ CMU)





# Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
  - diameter  $\sim O(\log N)$
  - diameter  $\sim O(\log \log N)$
- What is happening in real data?



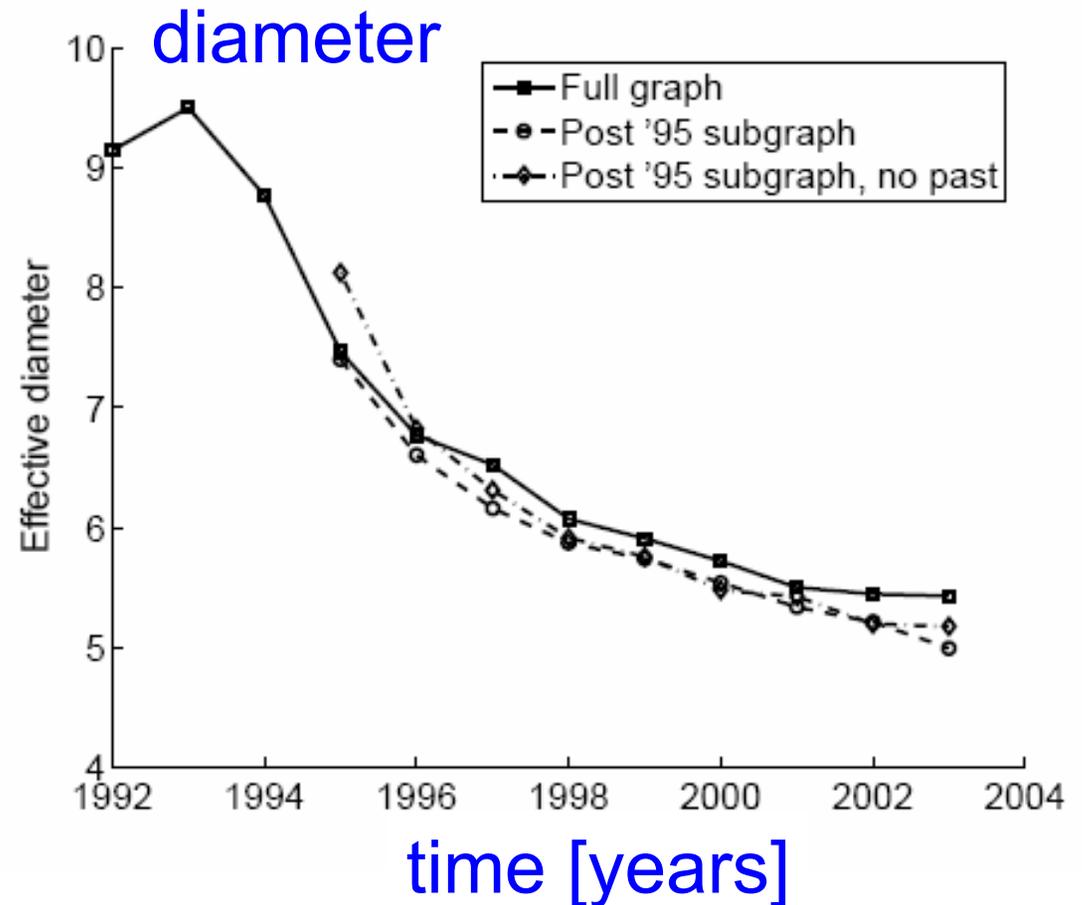
# Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
  - diameter  $\sim O(\log N)$
  - diameter  $\sim O(\log \log N)$
- What is happening in real data?
- Diameter **shrinks** over time



# Diameter – ArXiv citation graph

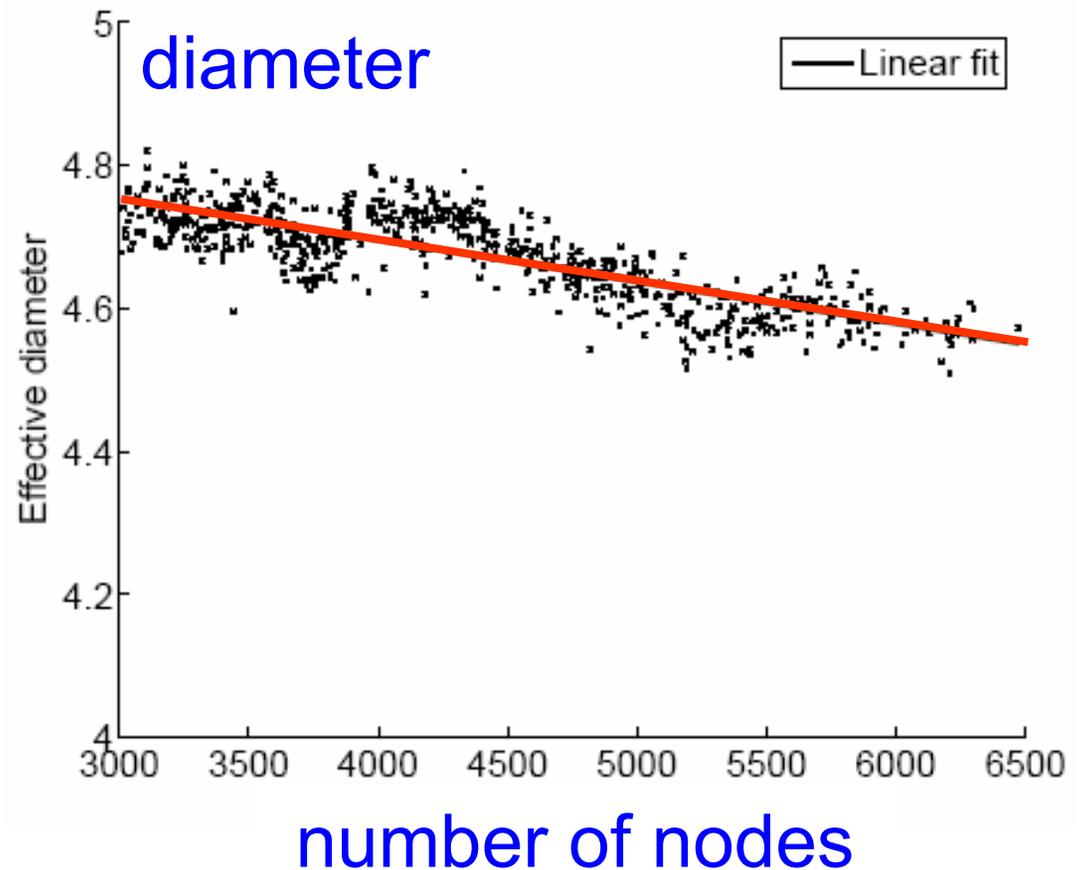
- Citations among physics papers
- 1992 – 2003
- One graph per year





# Diameter – “Autonomous Systems”

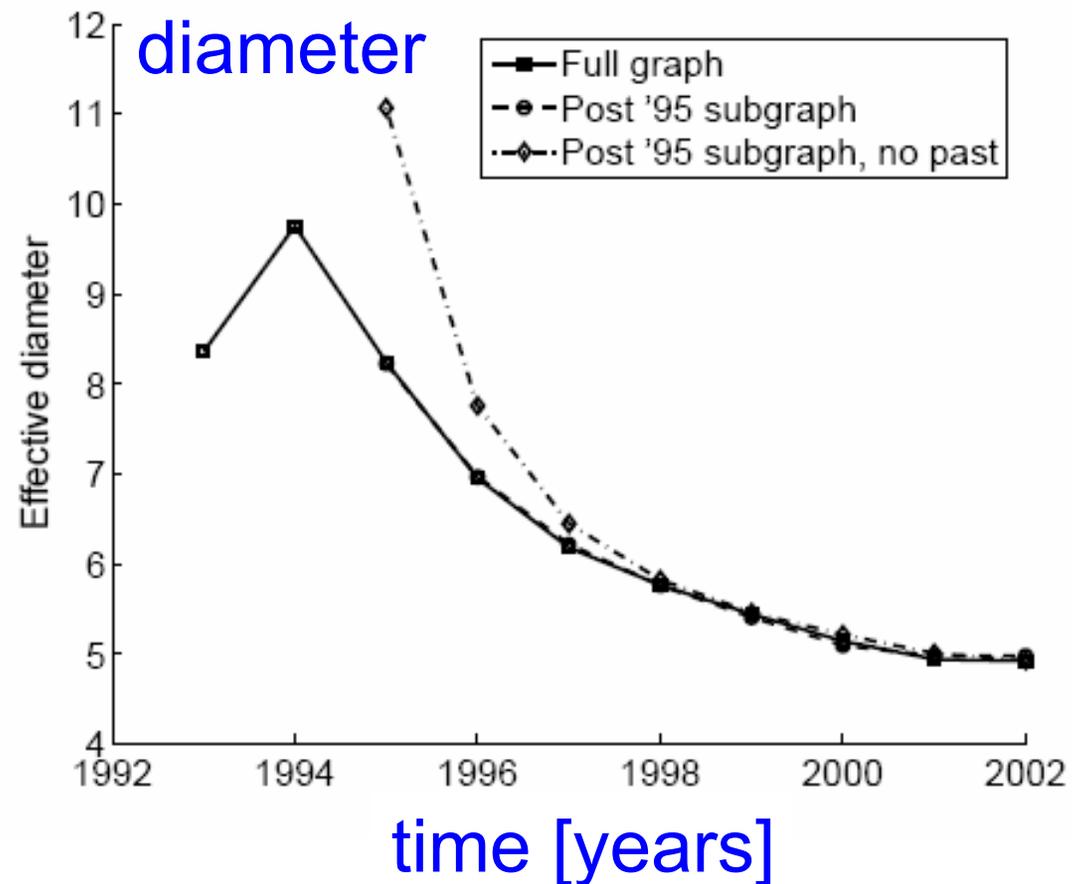
- Graph of Internet
- One graph per day
- 1997 – 2000





# Diameter – “Affiliation Network”

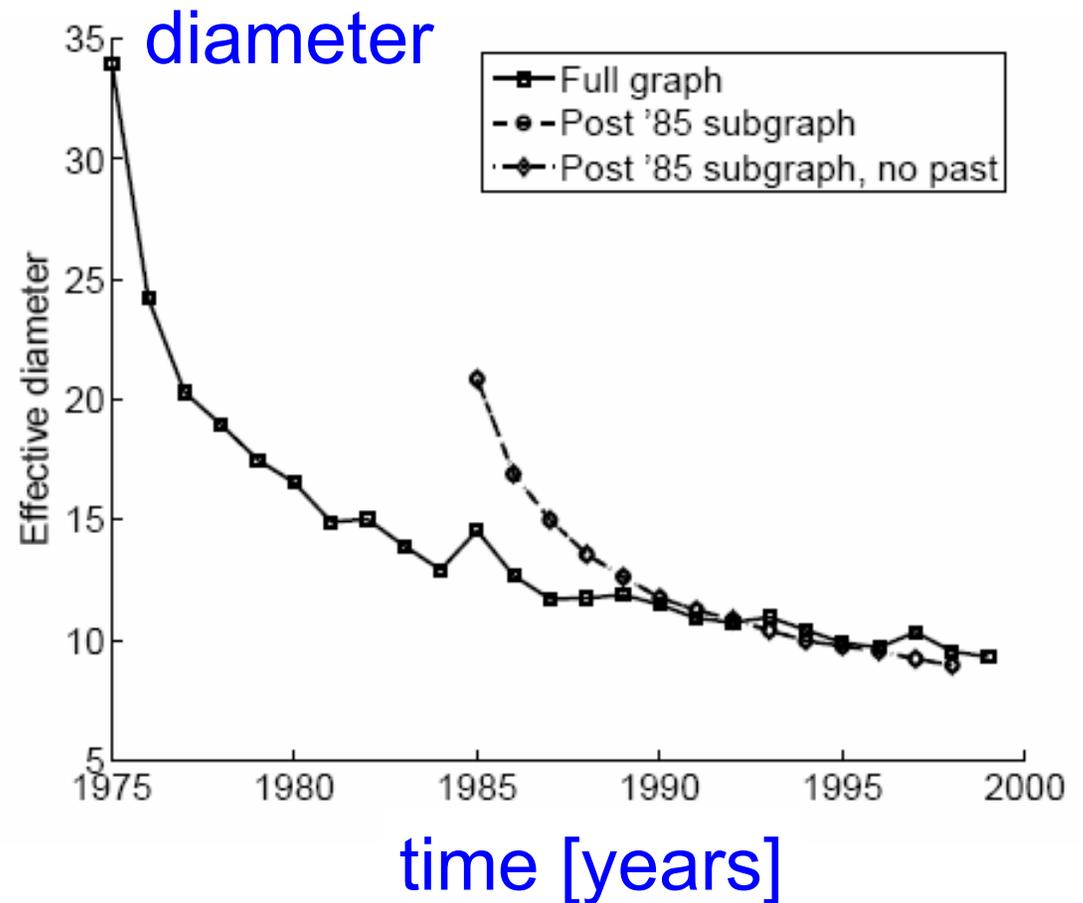
- Graph of collaborations in physics – authors linked to papers
- 10 years of data





# Diameter – “Patents”

- Patent citation network
- 25 years of data





# Temporal Evolution of the Graphs

- $N(t)$  ... nodes at time  $t$
- $E(t)$  ... edges at time  $t$
- Suppose that
$$N(t+1) = 2 * N(t)$$
- Q: what is your guess for
$$E(t+1) =? 2 * E(t)$$



# Temporal Evolution of the Graphs

- $N(t)$  ... nodes at time  $t$
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- Suppose that

$$N(t+1) = 2 * N(t)$$

- Q: what is your guess for

$$E(t+1) = \text{?} * E(t)$$

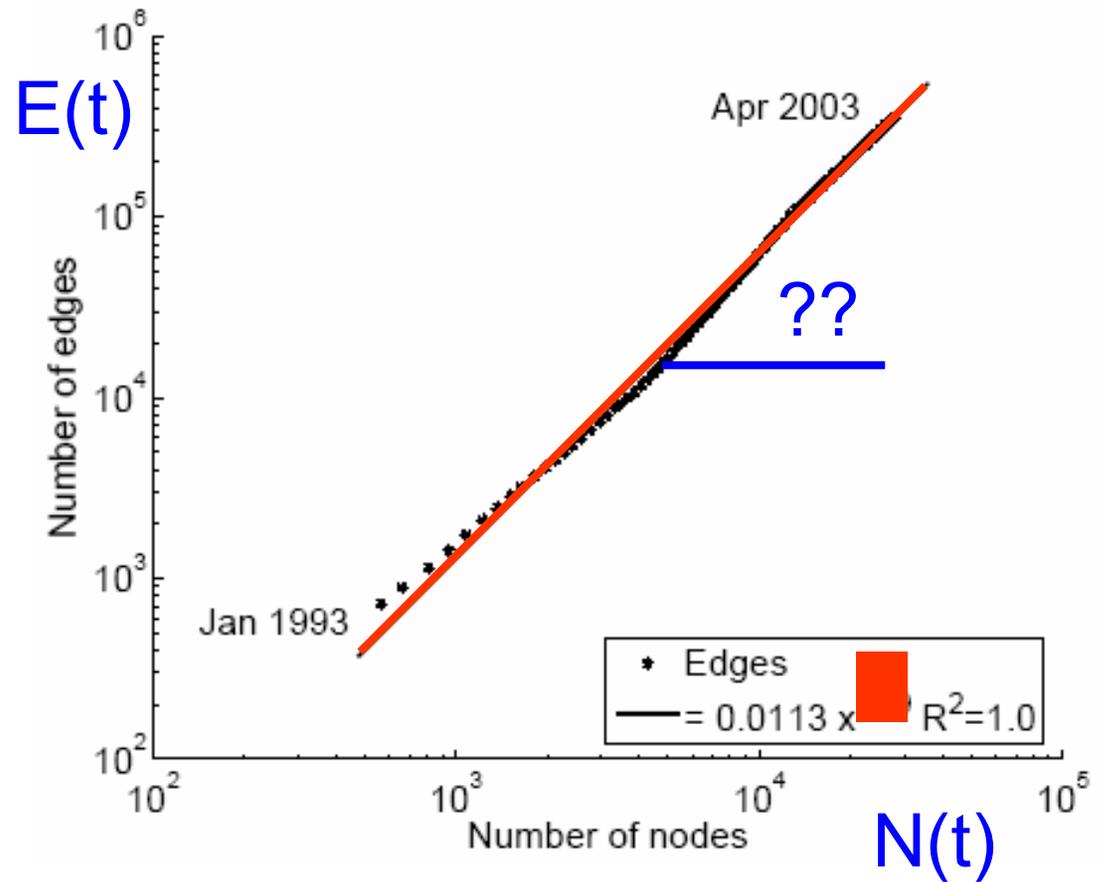
- A: over-doubled!

– But obeying the ‘‘Densification Power Law’’



# Densification – Physics Citations

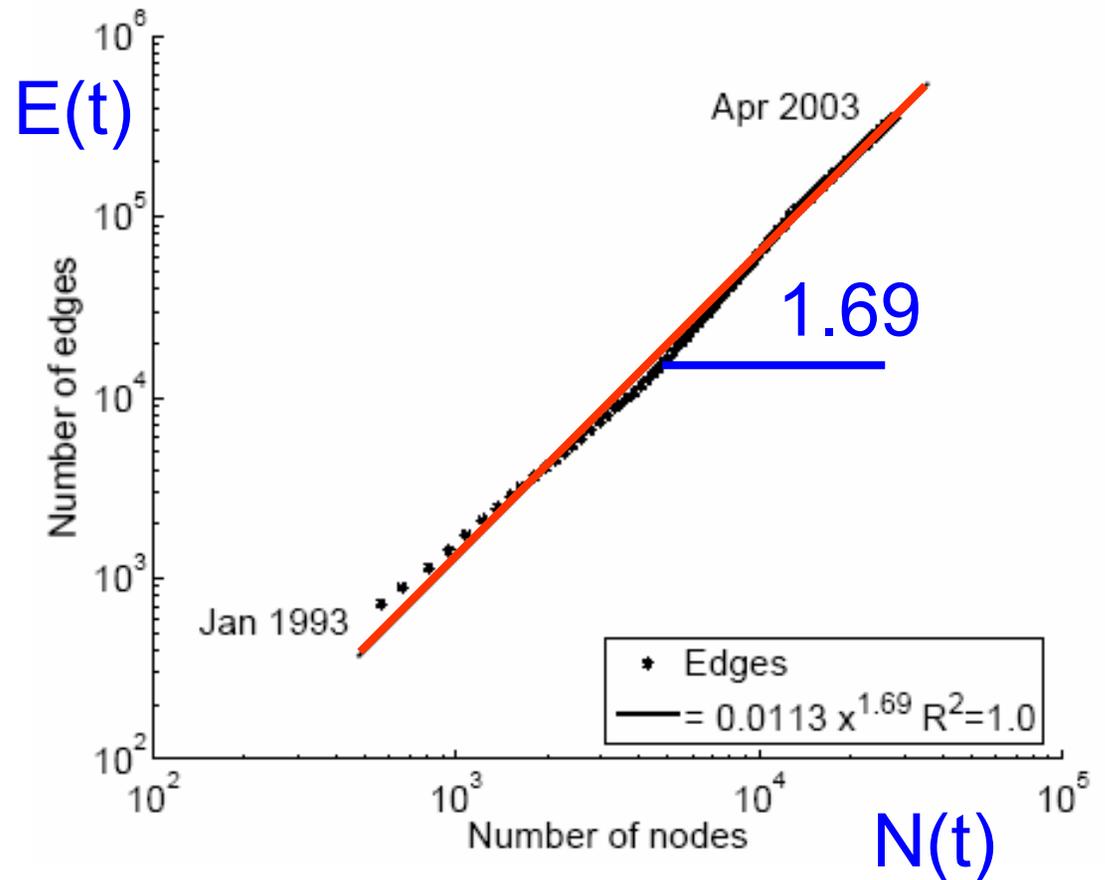
- Citations among physics papers
- 2003:
  - 29,555 papers, 352,807 citations





# Densification – Physics Citations

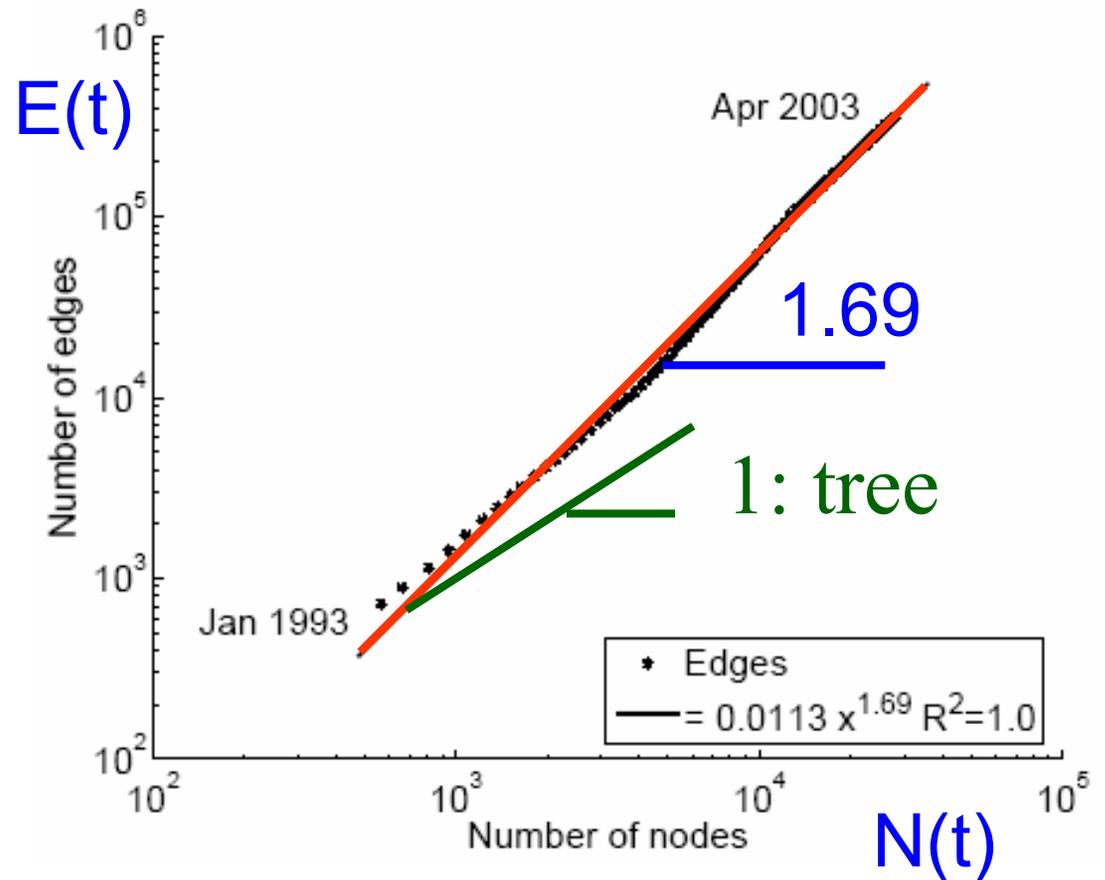
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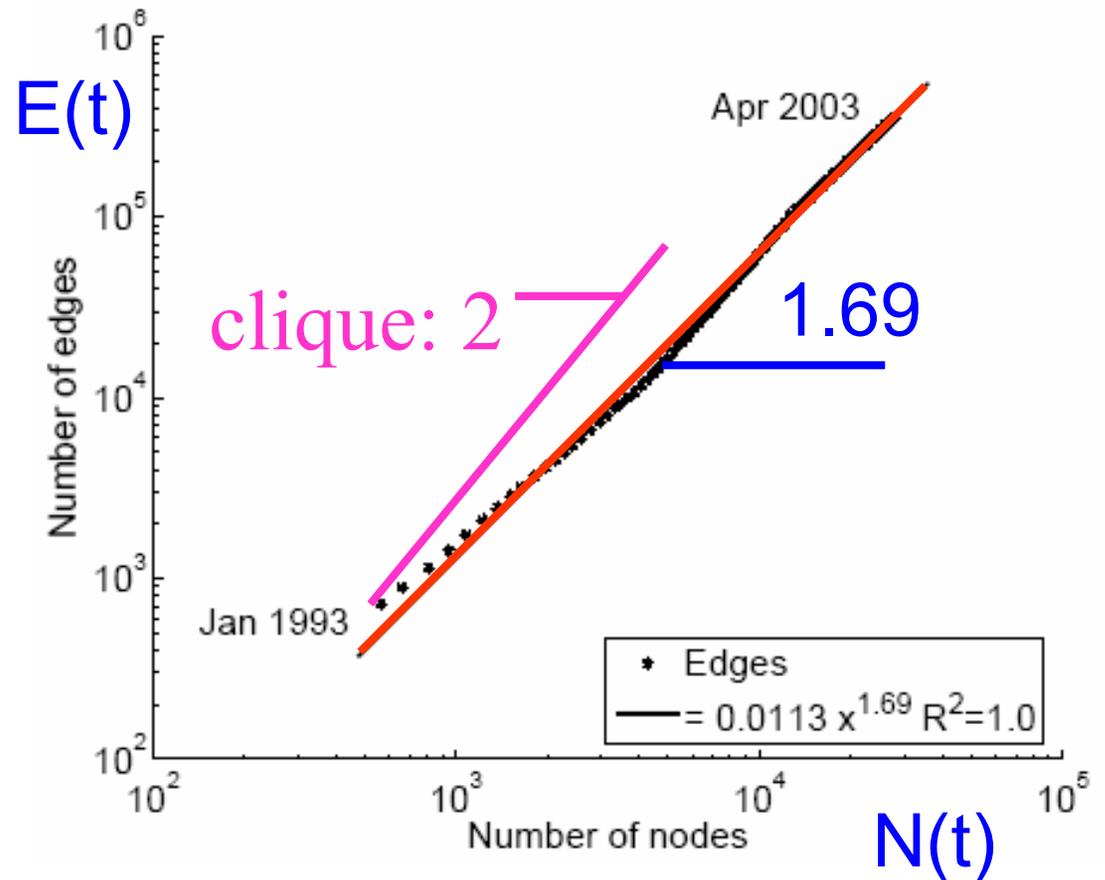
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# Densification – Physics Citations

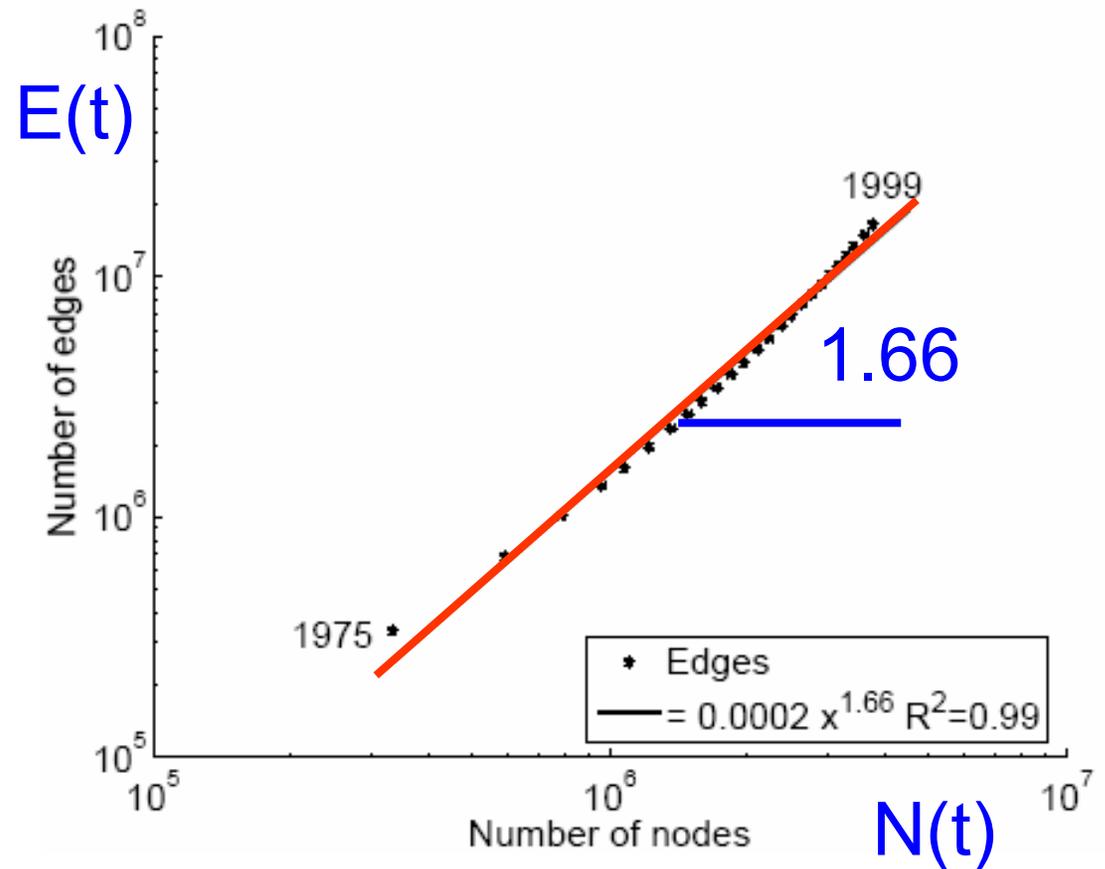
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# Densification – Patent Citations

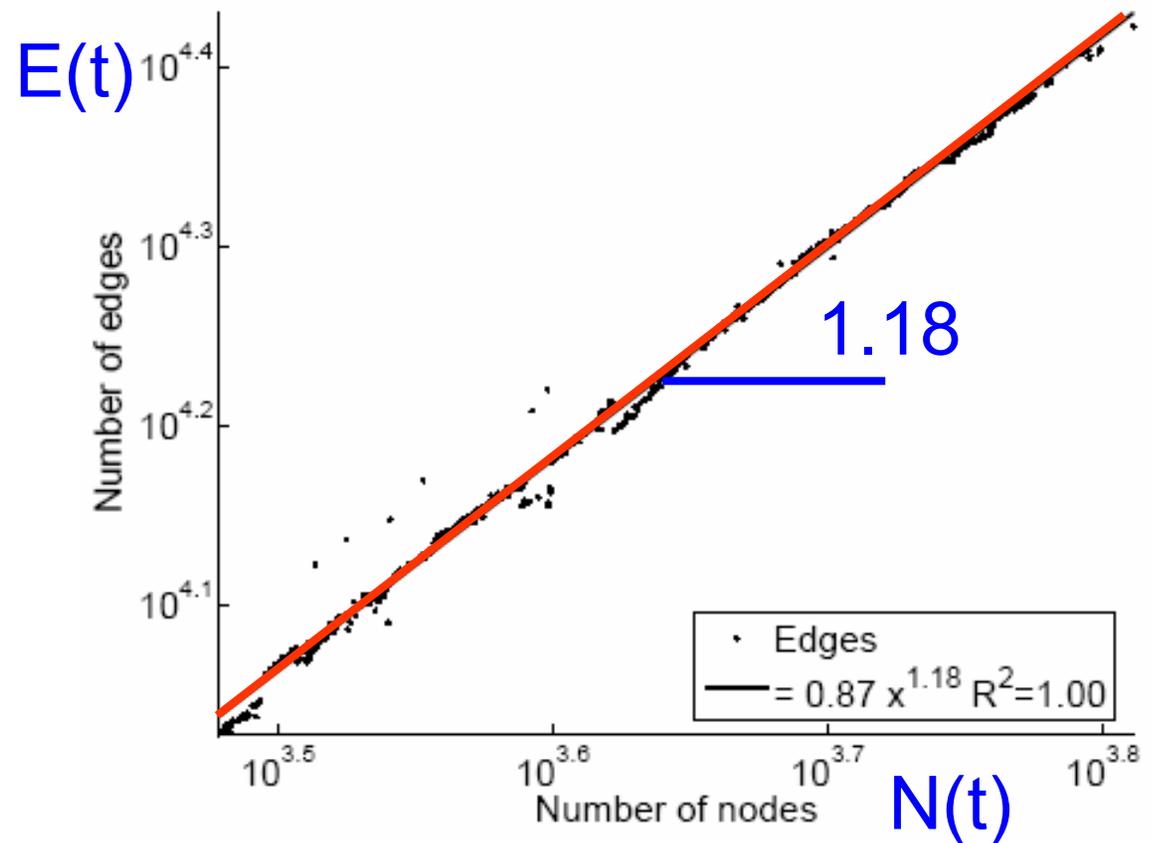
- Citations among patents granted
- 1999
  - 2.9 million nodes
  - 16.5 million edges
- Each year is a datapoint





# Densification – Autonomous Systems

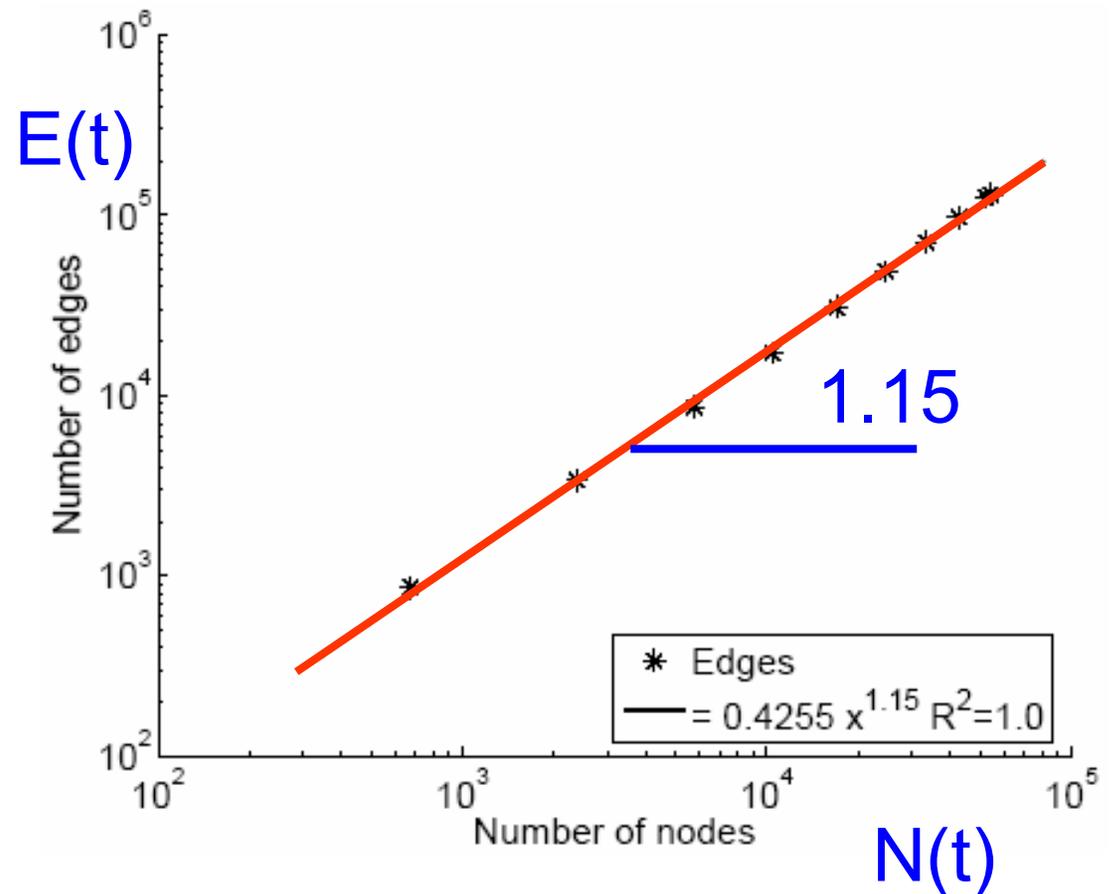
- Graph of Internet
- 2000
  - 6,000 nodes
  - 26,000 edges
- One graph per day





# Densification – Affiliation Network

- Authors linked to their publications
- 2002
  - 60,000 nodes
    - 20,000 authors
    - 38,000 papers
  - 133,000 edges





# Motivation

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- ✓ Problem#1: How do real graphs look like?
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## TOOLS

- Problem#4: Who is the ‘master-mind’?
- Problem#5: Fraud detection



## Problem#3: Generation

- Given a growing graph with count of nodes  $N_1$ ,  $N_2$ , ...
- Generate a realistic sequence of graphs that will obey all the patterns



# Problem Definition

- Given a growing graph with count of nodes  $N_1, N_2, \dots$
- Generate a realistic sequence of graphs that will obey all the patterns
  - Static Patterns
    - Power Law Degree Distribution
    - Power Law eigenvalue and eigenvector distribution
    - Small Diameter
  - Dynamic Patterns
    - Growth Power Law
    - Shrinking/Stabilizing Diameters

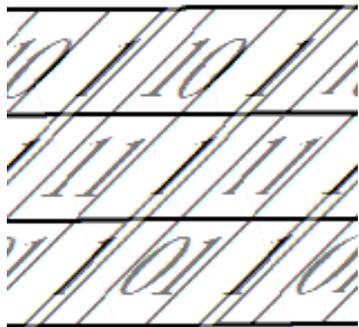
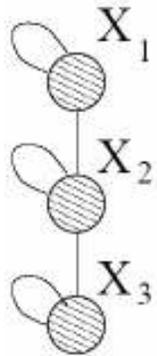


# Problem Definition

- Given a growing graph with count of nodes  $N_1, N_2, \dots$
- Generate a realistic sequence of graphs that will obey all the patterns
- **Idea: Self-similarity**
  - Leads to power laws
  - Communities within communities
  - ...



# Kronecker Product – a Graph



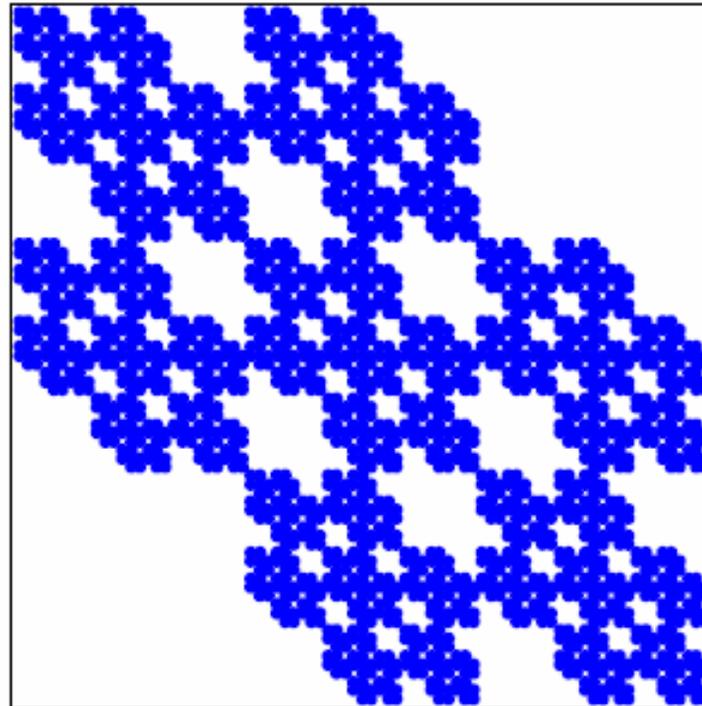
$G_1$

Adjacency matrix



# Kronecker Product – a Graph

- Continuing multiplying with  $G_1$  we obtain  $G_4$  and so on ...



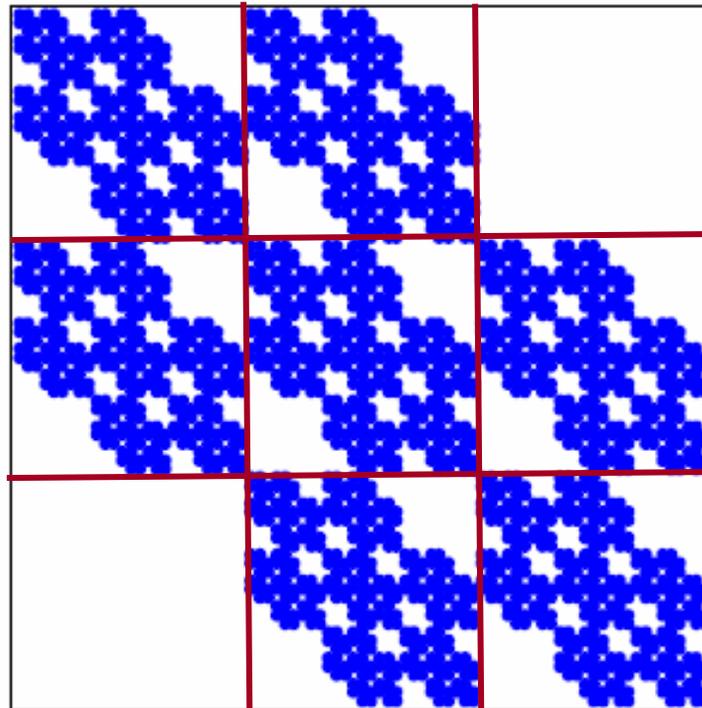
$G_4$  adjacency matrix

C. Faloutsos



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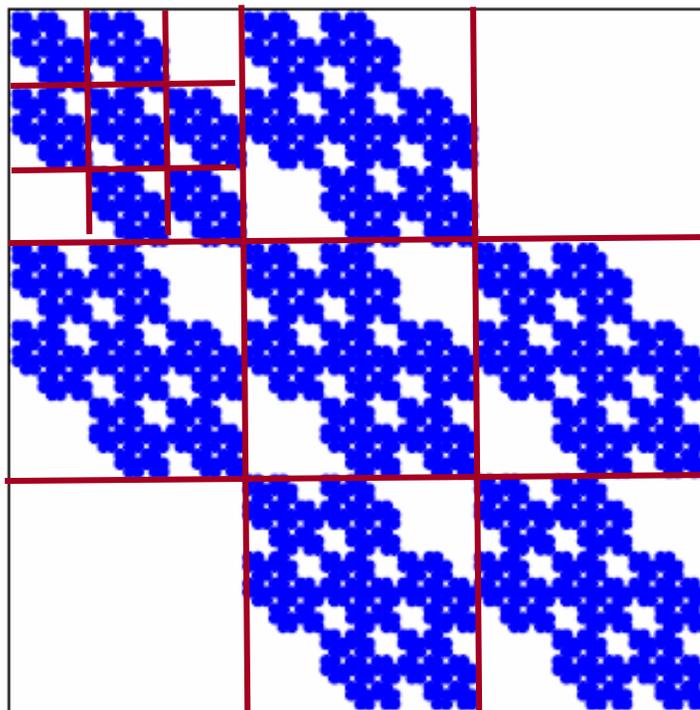
$G_4$  adjacency matrix

C. Faloutsos



# Kronecker Product – a Graph

- Continuing multiplying with  $G_1$  we obtain  $G_4$  and so on ...



$G_4$  adjacency matrix

C. Faloutsos



## Properties:

- We can PROVE that
  - Degree distribution is multinomial  $\sim$  power law
  - Diameter: constant
  - Eigenvalue distribution: multinomial
  - First eigenvector: multinomial
- See [Leskovec+, PKDD'05] for proofs



# Problem Definition

- Given a growing graph with nodes  $N_1, N_2, \dots$
- Generate a realistic sequence of graphs that will obey all the patterns
  - Static Patterns
    - ✓ Power Law Degree Distribution
    - ✓ Power Law eigenvalue and eigenvector distribution
    - ✓ Small Diameter
  - Dynamic Patterns
    - ✓ Growth Power Law
    - ✓ Shrinking/Stabilizing Diameters
- First and only generator for which we can **prove** all these properties



## (Q: how to fit the parm's?)

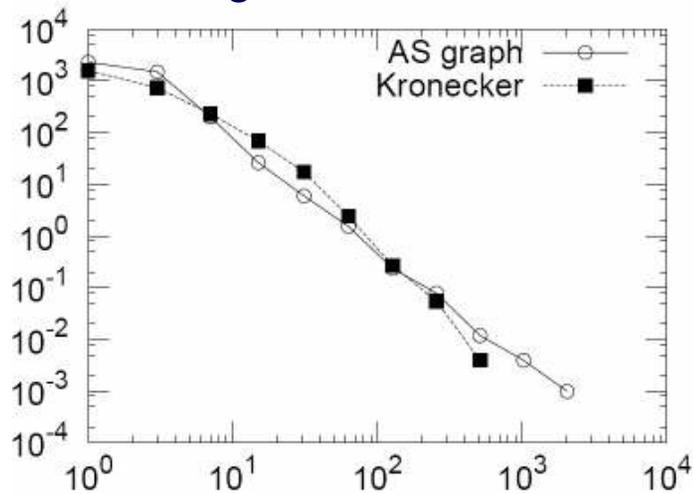
A:

- Stochastic version of Kronecker graphs +
- Max likelihood +
- Metropolis sampling
- [Leskovec+, ICML'07]

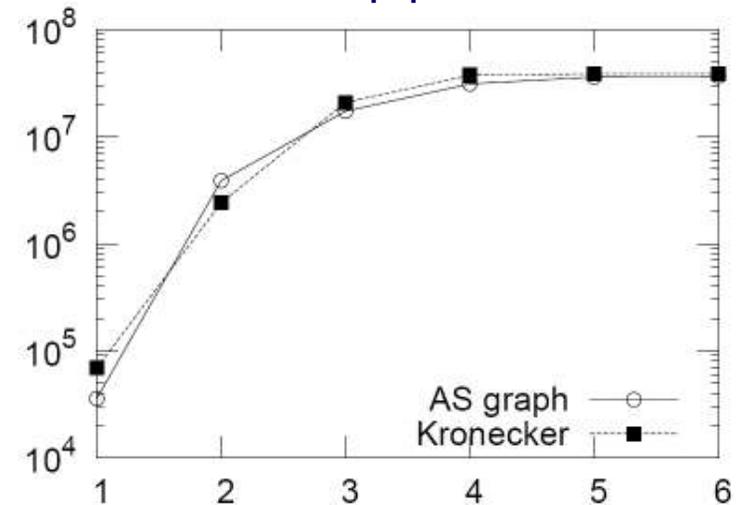


# Experiments on real AS graph

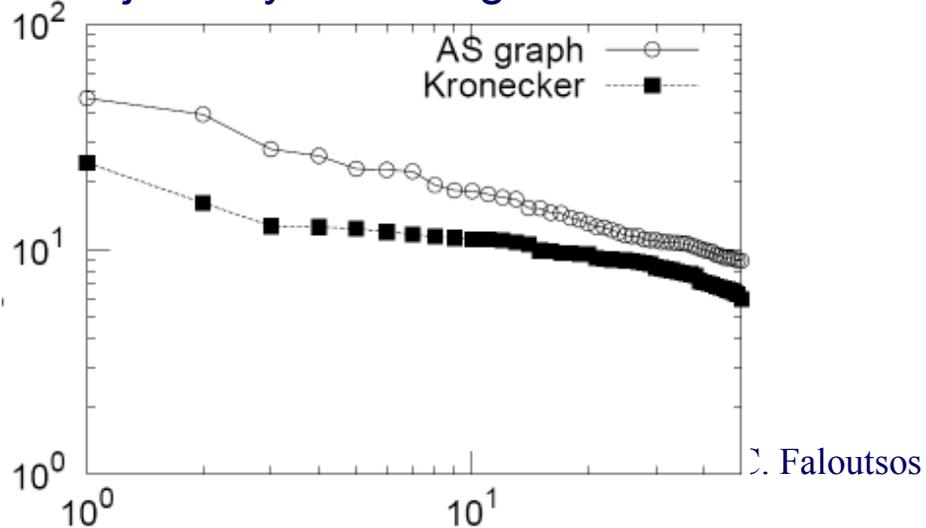
### Degree distribution



### Hop plot

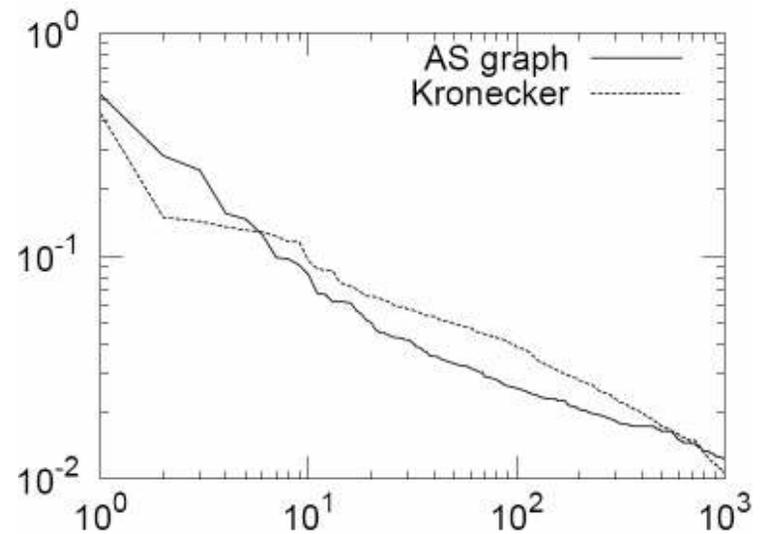


### Adjacency matrix eigen values



∪ Faloutsos

### Network value





# Conclusions

- Kronecker graphs have:
  - All the **static** properties
    - ✓ Heavy tailed degree distributions
    - ✓ Small diameter
    - ✓ Multinomial eigenvalues and eigenvectors
  - All the **temporal** properties
    - ✓ Densification Power Law
    - ✓ Shrinking/Stabilizing Diameters
  - We can formally **prove** these results



# Motivation

Data mining: ~ find patterns (rules, outliers)

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## TOOLS

- ➔ Problem#4: Who is the ‘master-mind’?
- Problem#5: Fraud detection



## Problem#4: MasterMind – ‘CePS’

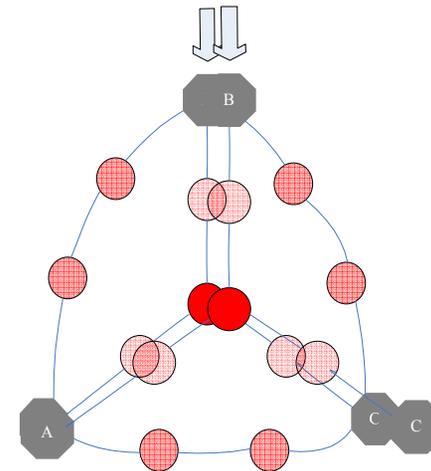
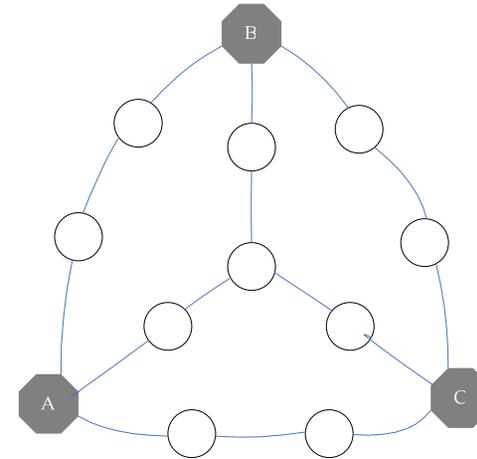
- w/ Hanghang Tong,  
KDD 2006
- htong <at> cs.cmu.edu





# Center-Piece Subgraph(Ceps)

- **Given**  $Q$  query nodes
- **Find** Center-piece ( $\leq b$  )
- **App.**
  - Social Networks
  - Law Enforcement, ...
- **Idea:**
  - Proximity  $\rightarrow$  random walk with restarts





# Case Study: AND query

R. Agrawal

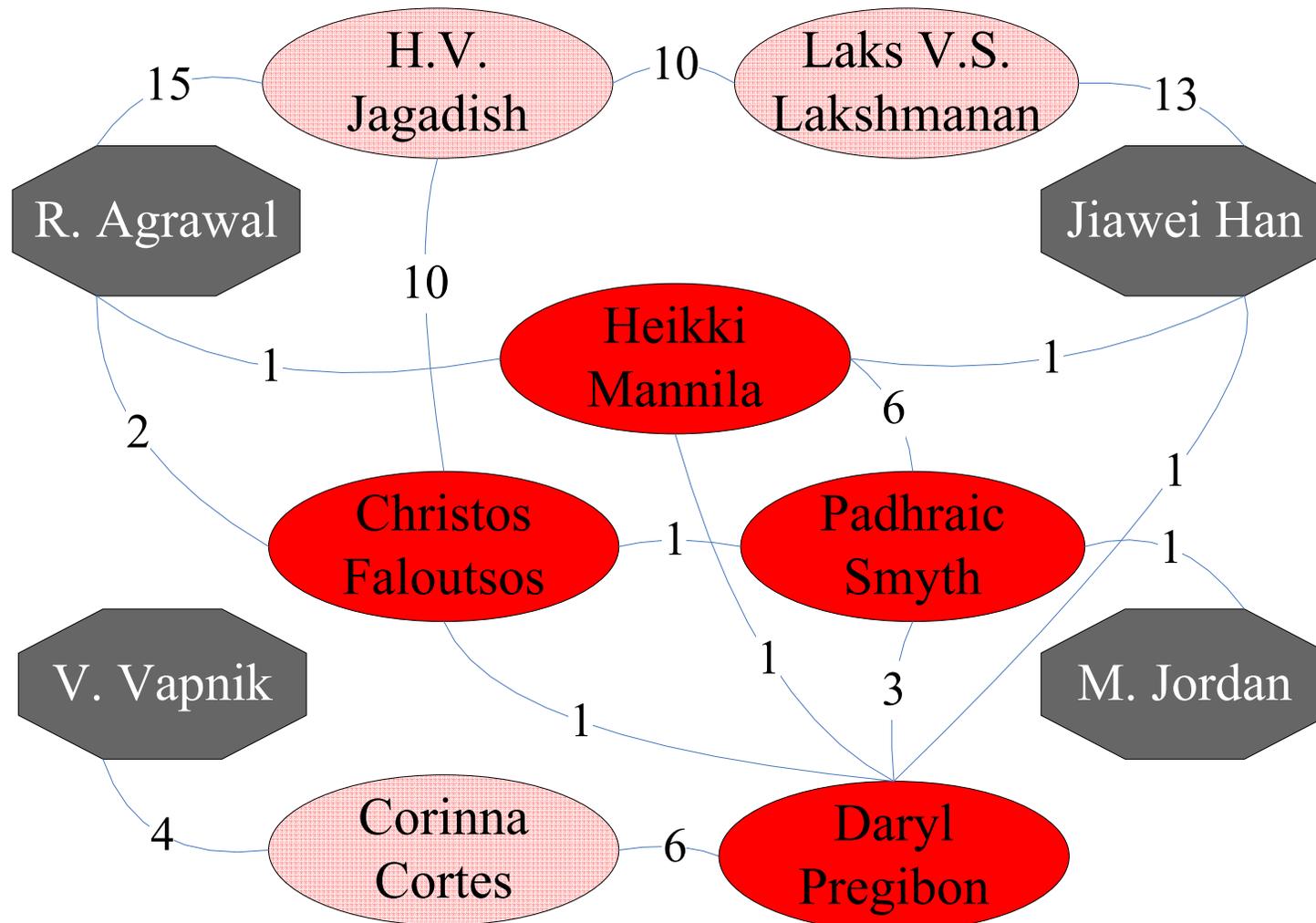
Jiawei Han

V. Vapnik

M. Jordan

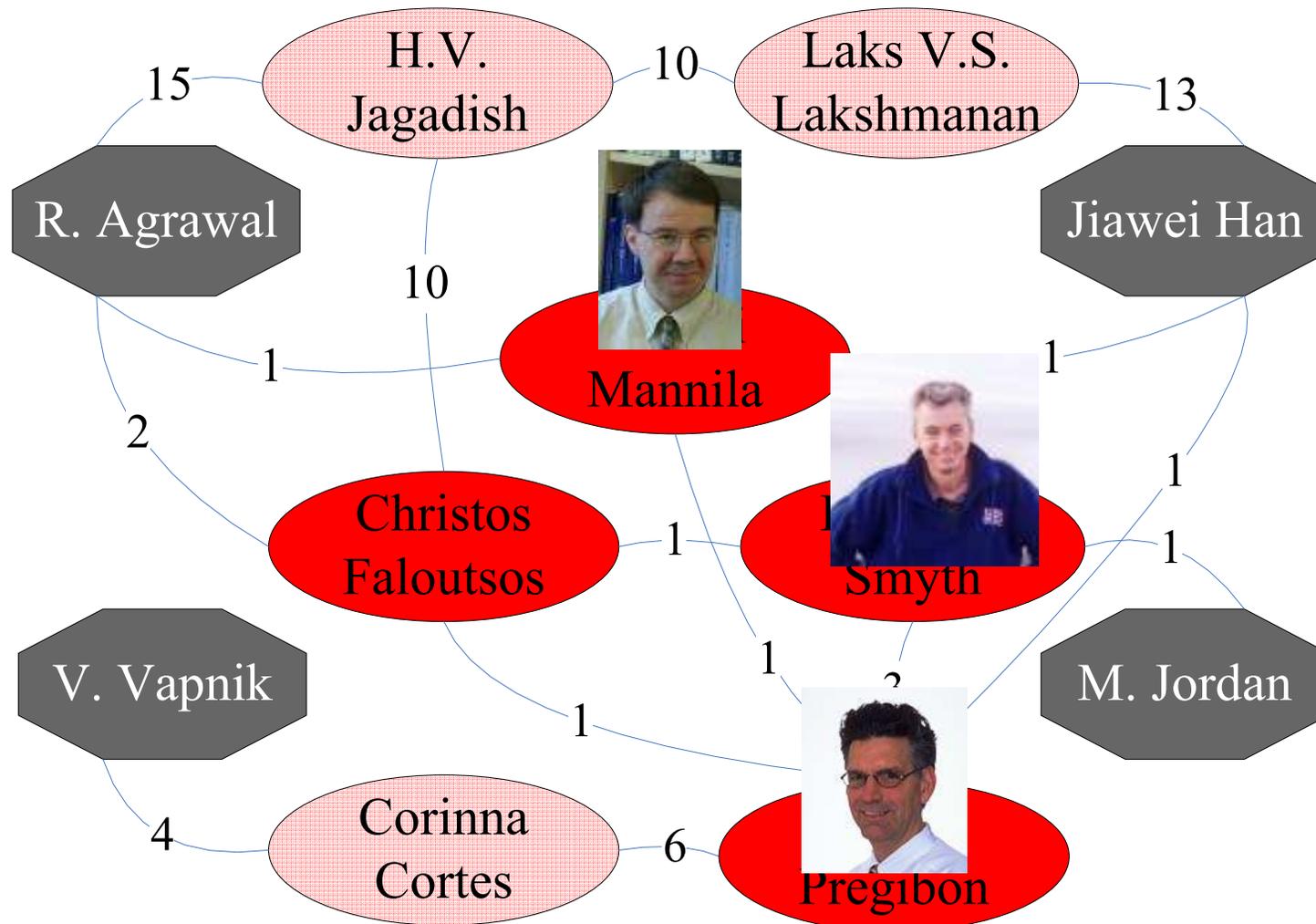


# Case Study: AND query





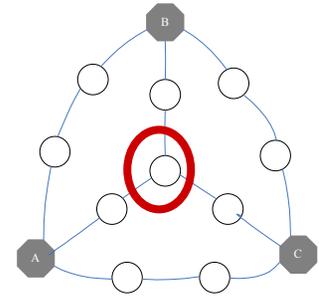
# Case Study: AND query





# Conclusions

- Q1:How to measure the importance?
- A1: RWR+K\_SoftAnd
- Q2:How to do it efficiently?
- A2:Graph Partition (Fast CePS)
  - ~90% quality
  - 150x speedup (ICDM'06)





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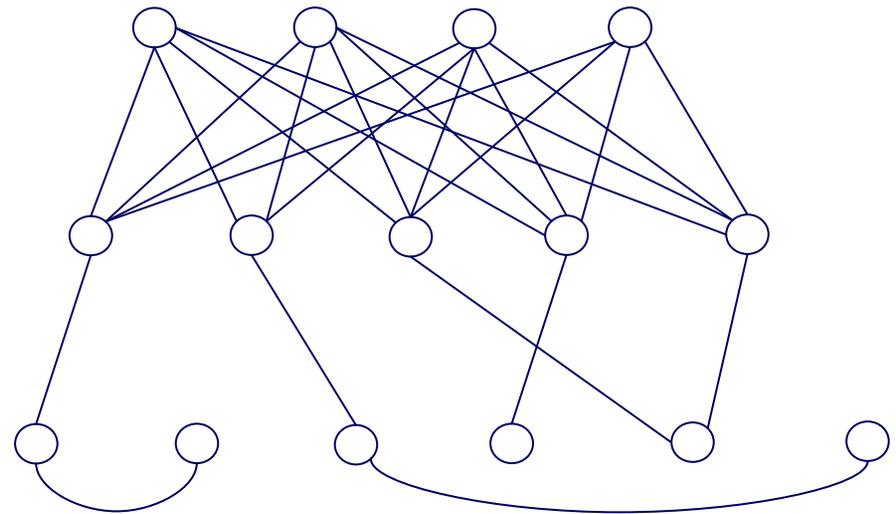
- ✓ Problem#4: Who is the ‘master-mind’?
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# E-bay Fraud detection



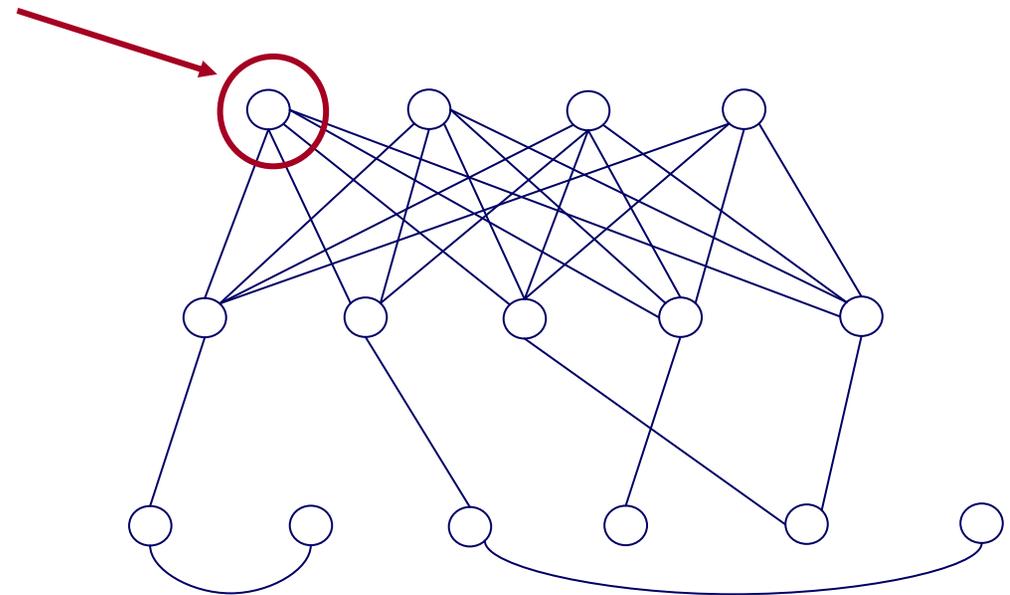
w/ Polo Chau &  
Shashank Pandit, CMU





# E-bay Fraud detection

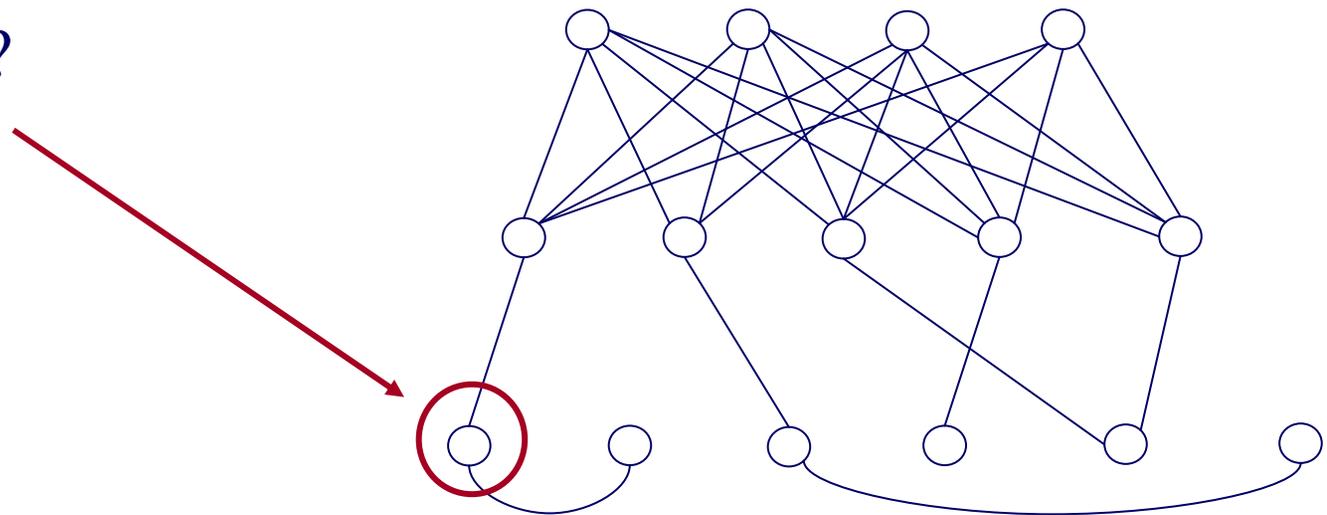
- lines: positive feedbacks
- would you buy from him/her?





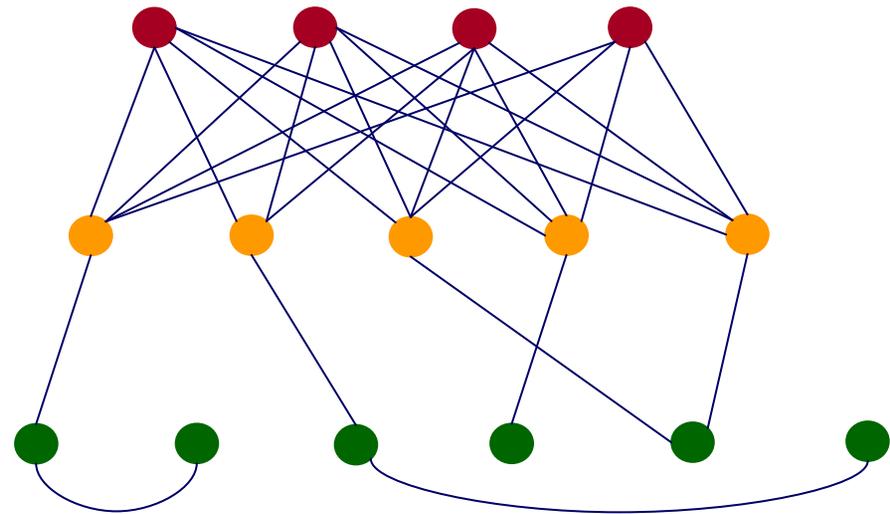
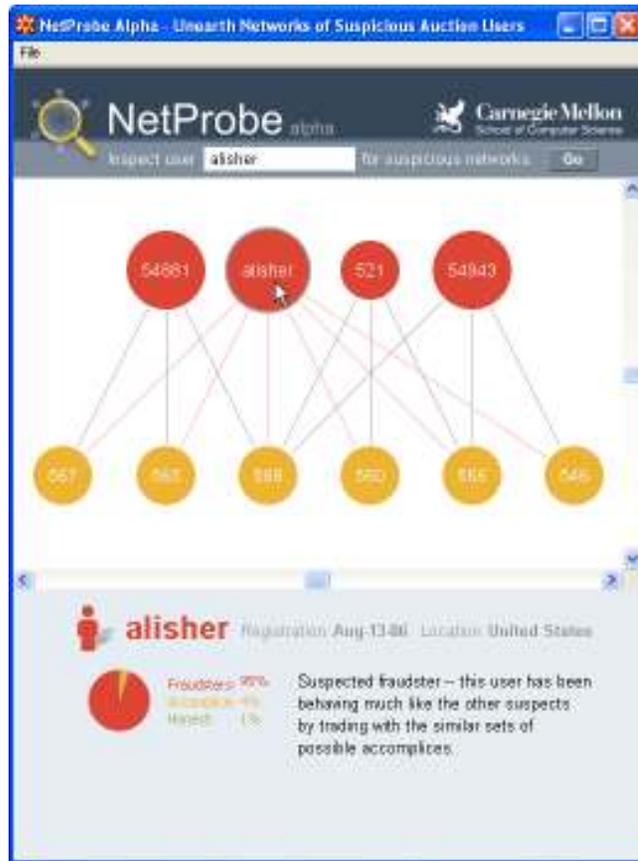
# E-bay Fraud detection

- lines: positive feedbacks
- would you buy from him/her?
- or him/her?





# E-bay Fraud detection - NetProbe





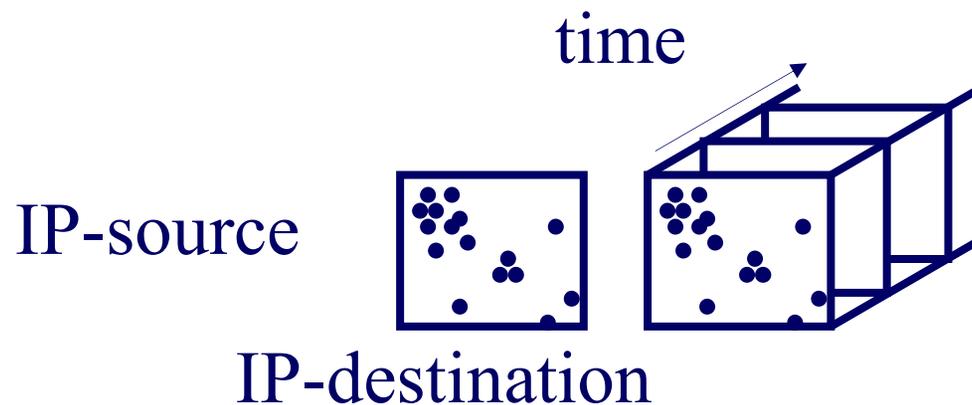
# OVERALL CONCLUSIONS

- Graphs pose a wealth of fascinating problems
- self-similarity and power laws work, when textbook methods fail!
- New patterns (shrinking diameter!)
- New generator: Kronecker



# Promising directions

- Reaching out
  - Sociology, epidemiology; physics, ++...
  - Computer networks, security, intrusion det.
  - Num. analysis (tensors)





## Promising directions – cont'd

- Scaling up, to Gb/Tb/Pb
  - Storage Systems
  - Parallelism (hadoop/map-reduce)



## E.g.: self-\* system @ CMU



- >200 nodes
- 40 racks of computing equipment
- 774kw of power.
- target: 1 PetaByte
- goal: self-correcting, self-securing, self-monitoring, self-  
...



# DM for Tera- and Peta-bytes

Two-way street:

<- DM can use such infrastructures to find patterns

-> DM can help such infrastructures become self-healing, self-adjusting, 'self-\*



# References

- Hanghang Tong, Christos Faloutsos, and Jia-Yu Pan *Fast Random Walk with Restart and Its Applications* ICDM 2006, Hong Kong.
- Hanghang Tong, Christos Faloutsos *Center-Piece Subgraphs: Problem Definition and Fast Solutions*, KDD 2006, Philadelphia, PA



# References

- Jure Leskovec, Jon Kleinberg and Christos Faloutsos [Graphs over Time: Densification Laws, Shrinking Diameters and Possible Explanations](#) KDD 2005, Chicago, IL. ("Best Research Paper" award).
- Jure Leskovec, Deepayan Chakrabarti, Jon Kleinberg, Christos Faloutsos [Realistic, Mathematically Tractable Graph Generation and Evolution, Using Kronecker Multiplication \(ECML/PKDD 2005\)](#), Porto, Portugal, 2005.



# References

- Jure Leskovec and Christos Faloutsos, *Scalable Modeling of Real Graphs using Kronecker Multiplication*, ICML 2007, Corvallis, OR, USA
- Shashank Pandit, Duen Horng (Polo) Chau, Samuel Wang and Christos Faloutsos [NetProbe: A Fast and Scalable System for Fraud Detection in Online Auction Networks](#) WWW 2007, Banff, Alberta, Canada, May 8-12, 2007.
- Jimeng Sun, Dacheng Tao, Christos Faloutsos [Beyond Streams and Graphs: Dynamic Tensor Analysis](#), KDD 2006, Philadelphia, PA



## References

- Jimeng Sun, Yinglian Xie, Hui Zhang, Christos Faloutsos. *Less is More: Compact Matrix Decomposition for Large Sparse Graphs*, SDM, Minneapolis, Minnesota, Apr 2007. [[pdf](#)]



# THANK YOU!

Contact info:

[www.cs.cmu.edu/~christos](http://www.cs.cmu.edu/~christos)

(w/ papers, datasets, code, etc)