Due: Thursday November 03, 2005

1. Let #SAT(ϕ) be the number of satisfying assignments in a Boolean formula ϕ. We have assumed in class that #SAT is complete for #P. This relies on the fact that there is a version of Cook's reduction from NP computations to SAT that is *parsimonious*. That is, given an NP machine N and an input string x, the parsimonious reduction will construct a Boolean formula ϕ such that $\#\mathrm{acc}_N(x) = \#\mathrm{SAT}(\phi)$.

   Look up Cook's reduction in your favorite source. Is the reduction as presented parsimonious? Justify your answer.

2. Define $\textsc{Span}_5(a_1, a_2, a_3, a_4, a_5)$ to be the number of distinct values in a_1, a_2, a_3, a_4, a_5. If #P is closed under $\textsc{Span}_5$ then given any five functions $f_1, \ldots, f_5 \in$ #P, there exists $g \in$ #P such that for all x,

   $$g(x) = \textsc{Span}_5(f_1(x), f_2(x), f_3(x), f_4(x), f_5(x)).$$

   Show that if #P is closed under $\textsc{Span}_5$, then UP = $\mathrm{C_=P}$.

   Note: We know UP $\subseteq \mathrm{C_=P} \subseteq$ PP. Also, UP = $\mathrm{C_=P}$ implies that UP = PP. Thus, by Theorem 5.6, #P is closed under $\textsc{Span}_5$ if and only if #P is closed under every polynomial-time computable operation.