Due: Thursday October 6, 2005

1. Extend the Karp-Lipton result and show that if co-NP $\subseteq$ NP/poly then PH collapses to Σ_3^{P}. I.e., show that

$$\overline{\mathrm{SAT}} \in \mathrm{NP/poly} \Longrightarrow \mathrm{NP}^{\mathrm{NP}^{\mathrm{NP}^{\mathrm{NP}}}} \subseteq \mathrm{NP}^{\mathrm{NP}^{\mathrm{NP}}}.$$

2. The result in Question 1 suggests that co-NP $\not\subseteq$ NP/poly, if we, for example, "believe" that PH does not collapse. The situation is very different for nondeterministic exponential time.

   Define NE as:

$$\mathrm{NE} = \bigcup_{c \geq 1} \mathrm{NTIME}\left[2^{cn}\right]$$

   and as usual define co-NE to be the complements:

$$\text{co-NE} = \{\overline{L} \mid L \in \mathrm{NE}\}.$$

   Show that co-NE $\subseteq$ NE/poly.