Due: February 13, 2003

1. Problem 1.23, parts c & d.

2. Problem 1.31.

3. Problems 1.34 and 1.35 in the textbook define for a language L when two strings $x, y \in \Sigma^*$ are *indistinguishable by* L (written $x \equiv_L y$.) We showed in class that the number of states in the smallest DFA that recognizes L is equal to the number of equivalence classes induced by $\equiv_L$. Although, one can define a minimal DFA from $\equiv_L$, the process is not constructive.

We can construct the set of distinguished states DIST of a DFA $M = (Q, \Sigma, \delta, q_0, F)$ as follows:

1. Let DIST be a list of all unordered pairs of states $\{p, q\}$ such that $p \in F$ and $q \notin F$.
2. For each pair of states $\{p, q\} \notin$ DIST, if there exists $a \in \Sigma$ such that $\{\delta(p, a), \delta(q, a)\} \in$ DIST, then add $\{p, q\}$ to DIST.
3. Repeat Step 2 if a new pair $\{p, q\}$ was added to DIST.

A careful implementation of this algorithm would result in a running time of $O(|Q|^2)$. After the construction of DIST, we can define an equivalence relation $\equiv_D$ on Q by:

$$p \equiv_D q \iff \{p, q\} \notin \text{DIST}.$$

Then we can construct a machine $M' = (Q', \Sigma, \delta', q_0', F')$ as follows. The set of states Q' is the set of equivalence classes induced by $\equiv_D$. We use $[p]$ to denote the equivalence class that contains p. The initial state $q_0' = [q_0]$ and the set of final states $F' = \{[p] \mid p \in F\}$. Finally, $\delta'([p], a) = [\delta(p, a)]$.

We claim that M' is a minimal DFA for $L = L(M)$. Justify this claim:

1. Argue that $L(M') = L(M)$.
2. In class we defined the equivalence relation $\equiv_M$ on Σ^* by:

   $$x \equiv_M y \iff \delta(q_0, x) = \delta(q_0, y),$$

   where by abuse of notation $\delta(q_0, x)$ is the state M enters after reading x. Argue that for all pairs of strings $x, y \in \Sigma^*$ that

   $$x \equiv_{M'} y \iff x \equiv_L y$$

   and that therefore M' has the smallest number of states.