CMSC 313 Lecture 20

- **More Karnaugh Map examples**
- **Quine-McCluskey (Tabular Reduction)**

Last Time

- Combinational logic components
- Introduction to Karnaugh Maps

Notes on K-maps

- Also works for POS
- Takes 2^n time for formulas with n variables
- Only optimizes two-level logic
  - Reduces number of terms, then number of literals in each term
- Assumes inverters are free
- Does not consider minimizations across functions
- Circuit minimization is generally a hard problem
- Quine-McCluskey can be used with more variables
- CAD tools are available if you are serious

Circuit Minimization is Hard

- Unix systems store passwords in encrypted form.
  - User types in x, system computes f(x) and looks for f(x) in a file.
- Suppose we us 64-bit passwords and I want to find the password x, such that f(x) = y. Let

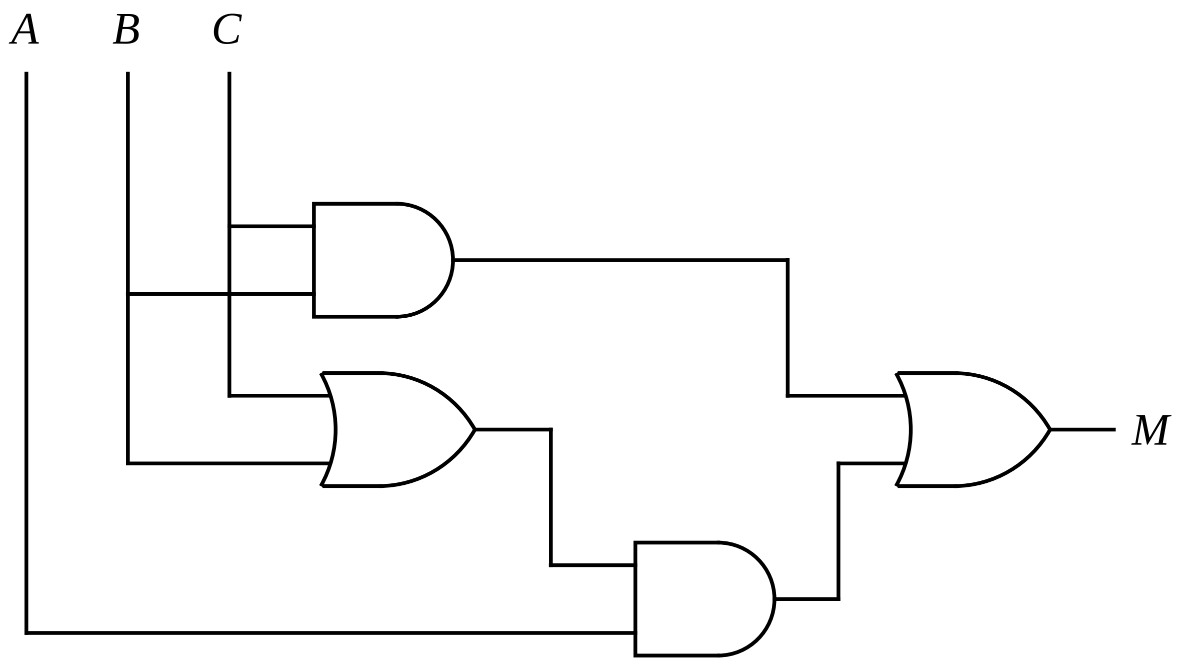
  $g_i(x)$ = 0 if f(x) = y and the ith bit of x is 0

  1 otherwise.

- If the ith bit of x is 1, then $g_i(x)$ outputs 1 for every x and has a very, very simple circuit.
- If you can simplify every circuit quickly, then you can crack passwords quickly.

3-Level Majority Circuit

- **K-Map Reduction results in a reduced two-level circuit (that is, AND followed by OR. Inverters are not included in the two-level count). Algebraic reduction can result in multi-level circuits with even fewer logic gates and fewer inputs to the logic gates.**



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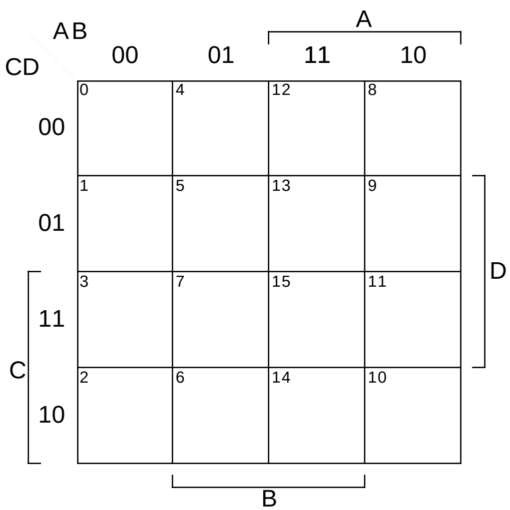
Karnaugh Maps

- **Implicant:** rectangle with 1, 2, 4, 8, 16 ... 1's
- **Prime Implicant:** an implicant that cannot be extended into a larger implicant
- **Essential Prime Implicant:** the only prime implicant that covers some 1
- **K-map Algorithm (not from M&H):**

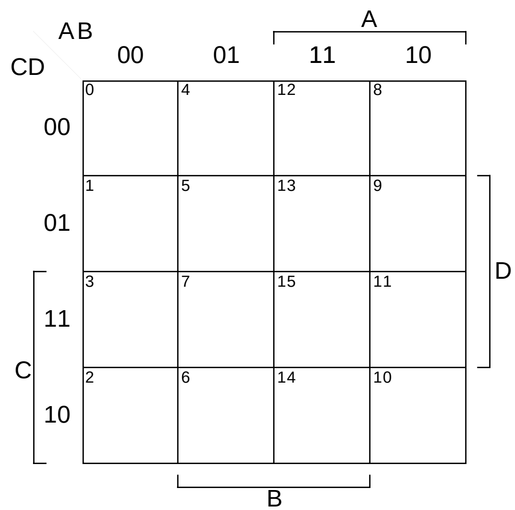
  1. Find ALL the prime implicants. Be sure to check every 1 and to use don't cares.

  2. Include all essential prime implicants.

  3. Try all possibilities to find the minimum cover for the remaining 1's.



A
AB
CD
00 01 11 10
00
01
11
10
0 4 12 8
1 5 13 9
3 7 15 11
2 6 14 10
C
D
B



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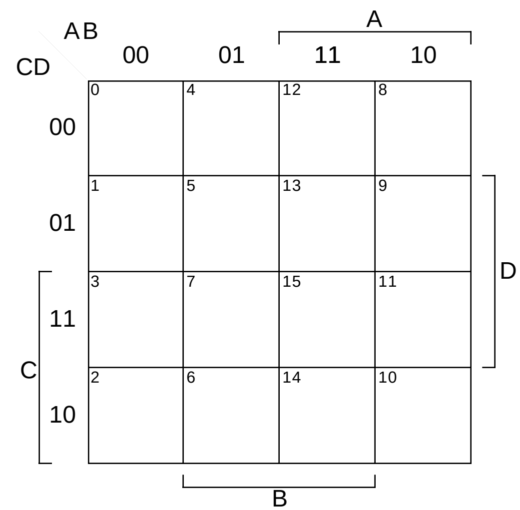

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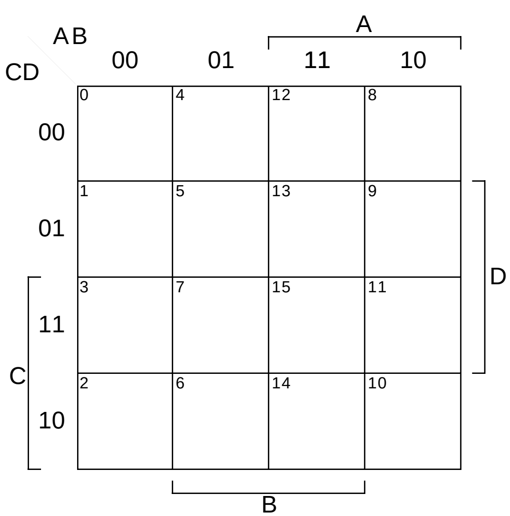

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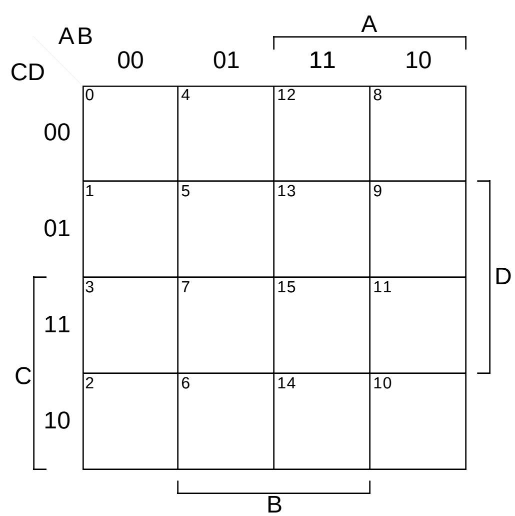

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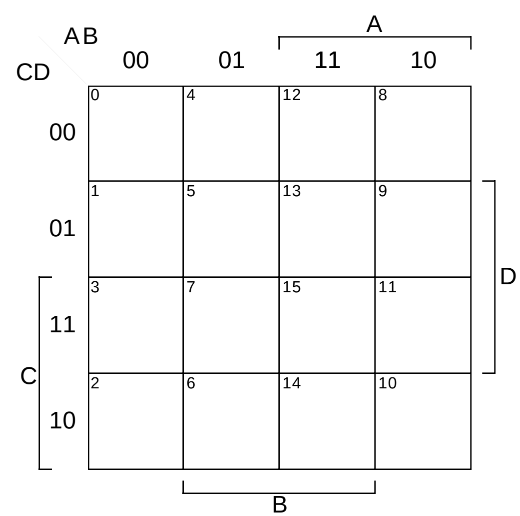

A
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C
D
B

Due: Thursday, November 18, 2003

1. (10 points) Question 3.9, page 96, Murdocca & Heuring
2. (10 points) Question 3.14, page 97, Murdocca & Heuring
3. (10 points) Question A.12, page 494, Murdocca & Heuring
4. (50 points) In the following, the notation $\sum m(x_1, \ldots, x_j)$ indicates a Boolean function that is the sum of the minterms $x_1, \ldots, x_j$, where x_i is the ith minterm in canonical ordering — i.e., the ith row of the truth table where the input values are ordered as binary numbers. Similarly,

$$\sum m(x_1, \ldots, x_j) + d(y_1, \ldots, y_k)$$

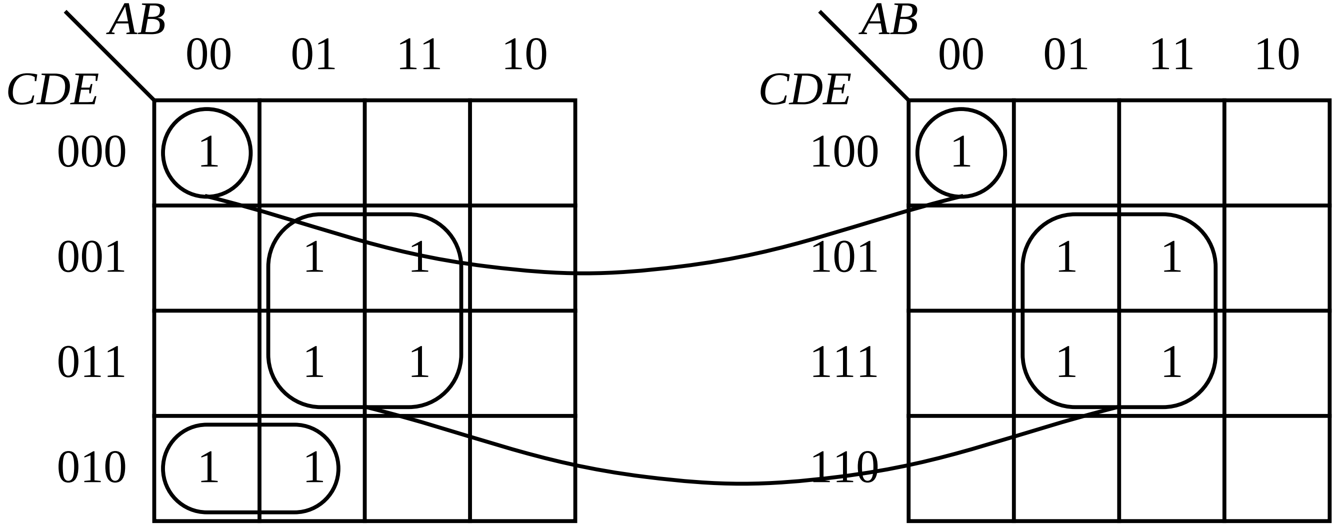
indicates a Boolean function that is the sum of the minterms $x_1, \ldots, x_j$ and whose values for rows $y_1, \ldots, y_k$ of the truth table are *don't cares.*

Minimize the following functions using Karnaugh maps. Then, write down a Boolean formula in sum-of-products or product-of-sums form for each function. Show your work (including the Karnaugh maps).

(a) $f(A, B, C, D) = \sum m(0, 1, 2, 8, 9, 14, 15)$
(b) $f(A, B, C, D) = \sum m(0, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 15)$
(c) $f(A, B, C, D) = \sum m(2, 3, 4, 5, 6, 7, 8, 9, 10, 13)$
(d) $f(A, B, C, D) = \sum m(4, 12, 13, 14, 15) + d(0, 3, 5, 8)$
(e) $f(A, B, C, D) = \sum m(0, 2, 4, 8, 10, 12, 13) + d(5, 14, 15)$

Five-Variable K-Map

- **Visualize two 4-variable K-maps stacked one on top of the other; groupings are made in three dimensional cubes.**



$$F = \overline{A}\,\overline{C}\,D\,\overline{E} + \overline{A}\,\overline{B}\,\overline{D}\,\overline{E} + B\,E$$

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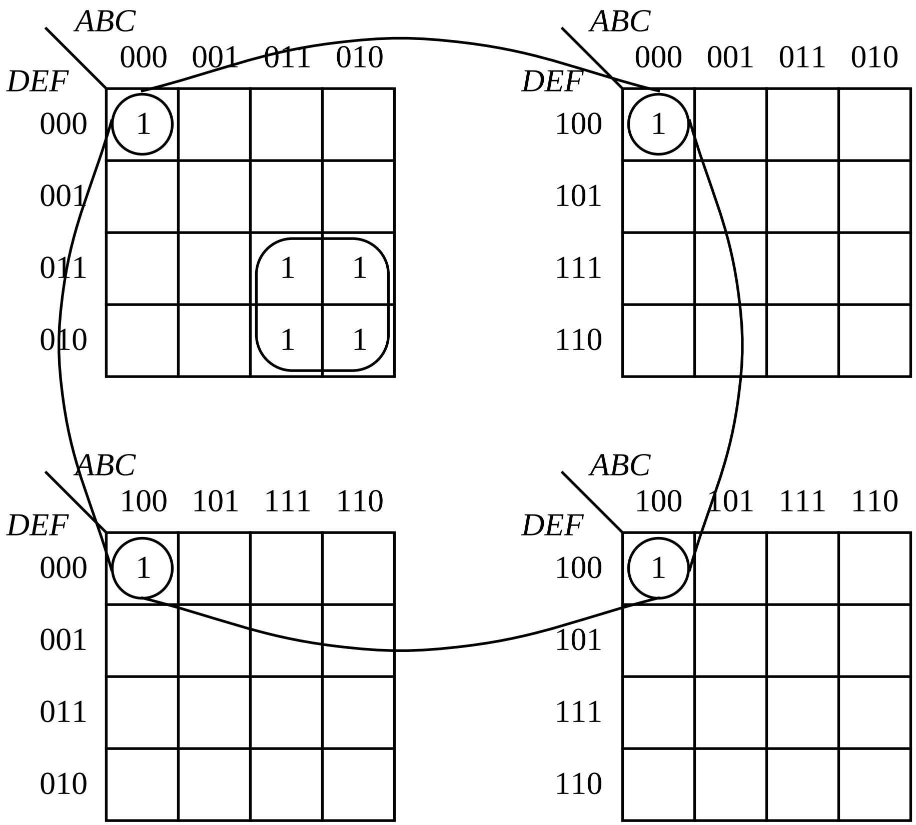
Six-Variable K-Map

- **Visualize four 4-variable K-maps stacked one on top of the other; groupings are made in three dimensional cubes.**



$$G = \overline{B}\,\overline{C}\,\overline{E}\,\overline{F} + \overline{A}\,B\,\overline{D}\,E$$

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Truth Table with Don't Cares

- **A truth table representation of a single function with don't cares.**

A	*B*	*C*	*D*	*F*
0	0	0	0	*d*
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	*d*
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	*d*

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Tabular (Quine-McCluskey) Reduction

- **Tabular reduction begins by grouping minterms for which *F* is nonzero according to the number of 1's in each minterm. Don't cares are considered to be nonzero.**
- **The next step forms a consensus (the logical form of a cross product) between each pair of adjacent groups for all terms that differ in only one variable.**

Initial setup

A	B	C	D	
0	0	0	0	√
0	0	0	1	√
0	0	1	1	√
0	1	0	1	√
0	1	1	0	√
1	0	1	0	√
0	1	1	1	√
1	0	1	1	√
1	1	0	1	√
1	1	1	1	√

(a)

After first reduction

A	B	C	D	
0	0	0	_	*
0	0	_	1	√
0	_	0	1	√
0	_	1	1	√
_	0	1	1	√
0	1	_	1	√
_	1	0	1	√
0	1	1	_	*
1	0	1	_	*
_	1	1	1	√
1	_	1	1	√
1	1	_	1	√

(b)

After second reduction

A	B	C	D	
0	_	_	1	*
_	_	1	1	*
_	1	_	1	*

(c)

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Table of Choice

- The prime implicants form a set that completely covers the function, although not necessarily minimally.
- A *table of choice* is used to obtain a minimal cover set.

Prime Implicants	Minterms						
	0001	0011	0101	0110	0111	1010	1101
0 0 0 _	√						
* 0 1 1 _				√	√		
* 1 0 1 _						√	
0 _ _ 1	√	√	√		√		
_ _ 1 1		√			√		
* _ 1 _ 1			√		√		√

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Reduced Table of Choice

- **In a reduced table of choice, the essential prime implicants and the minterms they cover are removed, producing the *eligible set*.**
- $F = \overline{A}BC + A\overline{B}C + BD + \overline{A}D$

Eligible Set		Minterms	
		0001	0011
X	0 0 0 _	√	
Y	0 _ _ 1	√	√
Z	_ _ 1 1		√

Set 1	**Set 2**
0 0 0 _	0 _ _ 1
_ _ 1 1	

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Multiple Output Truth Table

- **The power of tabular reduction comes into play for multiple functions, in which minterms can be shared among the functions.**

Minterm	A	B	C	F_0	F_1	F_2
m_0	0	0	0	1	0	0
m_1	0	0	1	0	1	0
m_2	0	1	0	0	0	1
m_3	0	1	1	1	1	1
m_4	1	0	0	0	1	0
m_5	1	0	1	0	0	0
m_6	1	1	0	0	1	1
m_7	1	1	1	1	1	1

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Multiple Output Table of Choice

$$F_0(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + BC$$

$$F_1(A,B,C) = \overline{A}C + A\overline{C} + BC$$

$$F_2(A,B,C) = B$$

Prime Implicants \ Min-terms		$F_0(A,B,C)$ m_0	m_3	m_7	$F_1(A,B,C)$ m_1	m_3	m_4	m_6	m_7	$F_2(A,B,C)$ m_2	m_3	m_6	m_7
F_0	* 0 0 0	√											
F_1	* 0 _ 1				√	√							
F_1	* 1 _ 0						√	√					
F_2	* _ 1 _									√	√	√	√
$F_{1,2}$	1 1 _							√	√			√	√
$F_{0,1,2}$	* _ 1 1		√	√		√			√		√		√

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