CMSC 313 Lecture 17

- **Postulates & Theorems of Boolean Algebra**
- **Semiconductors**
- **CMOS Logic Gates**

Last Time

- Overview of second half of this course
- Logic gates & symbols
- Equivalence of Boolean functions, truth tables, Boolean formulas and combinational circuits
- Universality of NAND gates

Postulates of Boolean Algebra

- Commutative: $AB = BA$, $A + B = B + A$
- Associative: $(AB)C = A(BC)$, $(A + B) + C = A + (B + C)$
- Distributive: $A(B + C) = AB + AC$, $A + BC = (A + B)(A + C)$
- Identity: there exists 0 and 1 such that for all A,

  $$1A = A \quad \text{and} \quad 0 + A = A.$$

- Complement: for all A, there exists $\overline{A}$ such that

  $$A\overline{A} = 0 \quad \text{and} \quad A + \overline{A} = 1$$

  where 0 and 1 are the identity elements.

Some Theorems of Boolean Algebra

- **Zero and One:** $0A = 0, \quad 1 + A = 1$

- **Idempotence:** $AA = A, \quad A + A = A$

- **Involution:** $\overline{\overline{A}} = A$

- **DeMorgan's:** $\overline{AB} = \overline{A} + \overline{B}, \quad \overline{A + B} = \overline{A}\,\overline{B}$

- **Absorption:** $A(A + B) = A, \quad A + AB = A$

- **Consensus:** $AB + \overline{A}C + BC = AB + \overline{A}C,$
  $(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$

Theorems are derived from the postulates

- Idempotence:

$$\begin{aligned}
A &= A1 && \text{identity, commutative} \\
&= A(A+\overline{A}) && \text{complement} \\
&= AA + A\overline{A} && \text{distributive} \\
&= AA + 0 && \text{complement} \\
&= AA && \text{commutative, identity}
\end{aligned}$$

$$\begin{aligned}
A &= 0 + A && \text{identity} \\
&= (A\overline{A}) + A && \text{complement} \\
&= (A+A)(\overline{A}+A) && \text{distributive} \\
&= (A+A)1 && \text{commutative, complement} \\
&= A + A && \text{commutative, identity}
\end{aligned}$$

Theorems are derived from the postulates

- Zero and One:

$$\begin{aligned}
0A &= (A\overline{A})A && \text{complement}\\
&= A(A\overline{A}) && \text{commutative}\\
&= (AA)\overline{A} && \text{associative}\\
&= A\overline{A} && \text{idempotent}\\
&= 0 && \text{complement}
\end{aligned}$$

$$\begin{aligned}
1 + A &= (A + \overline{A}) + A && \text{complement}\\
&= A + (A + \overline{A}) && \text{commutative}\\
&= (A + A) + \overline{A} && \text{associative}\\
&= A + \overline{A} && \text{idempotent}\\
&= 1 && \text{complement}
\end{aligned}$$

Theorems are derived from the postulates

- Absorption:

$$
\begin{aligned}
A + AB &= A1 + AB && \text{identity, commutative} \\
&= A(1 + B) && \text{distributive} \\
&= A1 && \text{one} \\
&= A && \text{commutative, identity}
\end{aligned}
$$

$$
\begin{aligned}
A(A + B) &= AA + AB && \text{distributive} \\
&= A + AB && \text{idempotent} \\
&= A && \text{absorption}
\end{aligned}
$$

Proof by truth table?

- Absorption: $A + AB = A$

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

- Proof by truth table only applies to 0-1 Boolean Algebra.

- Proof by derivation from postulates holds for any Boolean Algebra.

A Boolean Algebra with 8 elements

- Elements are the subsets of $\{a, b, c\}$:

$$\emptyset,\ \{a\},\ \{b\},\ \{c\},\ \{a,b\},\ \{a,c\},\ \{b,c\},\ \{a,b,c\}$$

- Operations: $AB \longrightarrow A \cap B,\ \ A + B \longrightarrow A \cup B$.
- Union and intersection are commutative and associative.
- Union distributes over intersection: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
- Intersection distributes over union: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
- Identity: $0 = \emptyset,\ \ 1 = \{a, b, c\},\ \ \{a, b, c\} \cap A = A$ and $\emptyset \cup A = A$.
- Complement: $\overline{A} = \{a, b, c\} - A,\ \ A \cap \overline{A} = \emptyset$ and $A \cup \overline{A} = \{a, b, c\}$.
- All postulates hold. Therefore, all derived theorems also hold.

Simplifying MAJ3

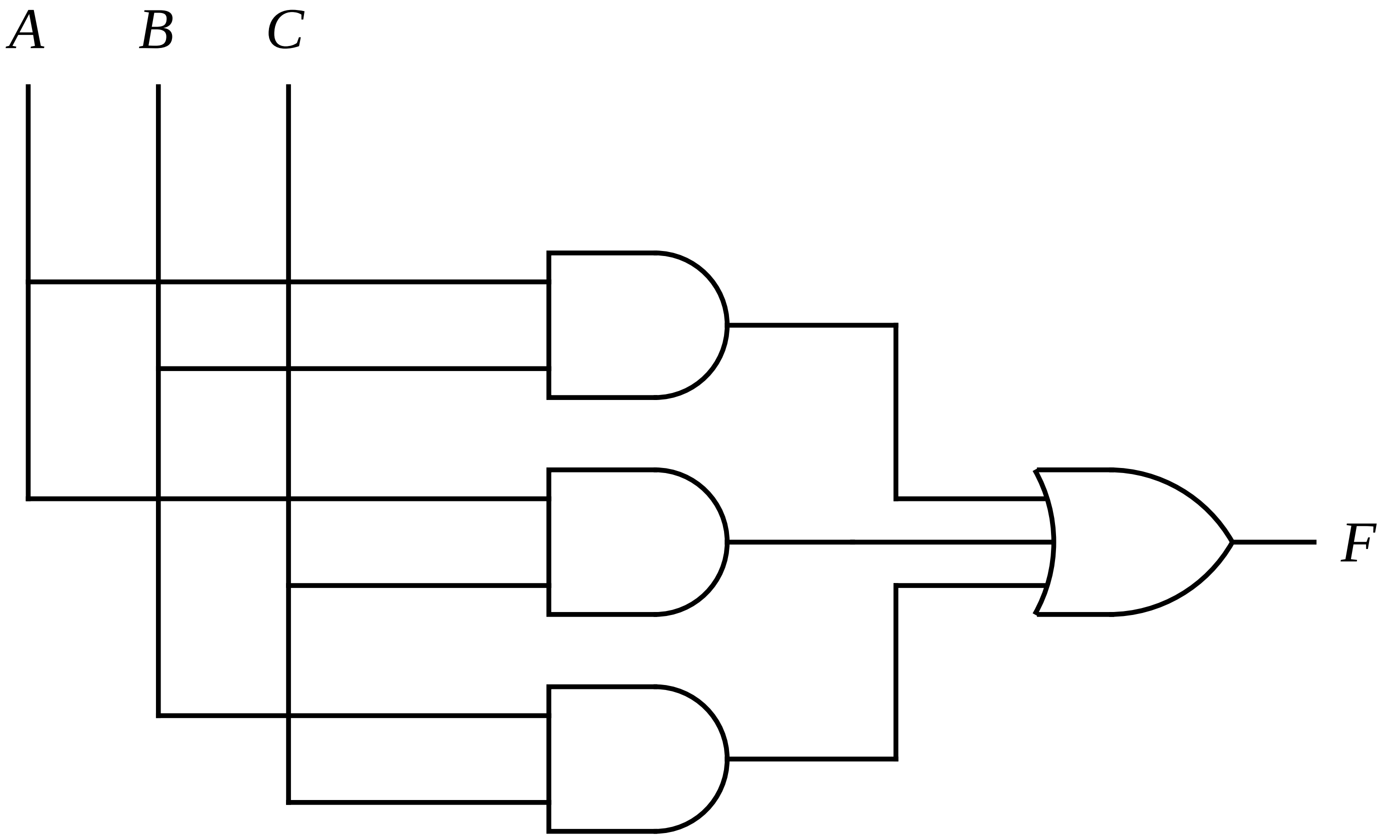
- Simplify MAJ3 using the postulates and theorems of Boolean Algebra

$$
\begin{aligned}
&\mathrm{MAJ3}(A, B, C) & \\
&= \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC & \text{SOP form} \\
&= \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC + ABC + ABC & \text{idempotent} \\
&= \overline{A}BC + ABC + A\overline{B}C + ABC + AB\overline{C} + ABC & \text{commutative} \\
&= (\overline{A} + A)BC + (\overline{B} + B)AC + (\overline{C} + C)AB & \text{distributive} \\
&= 1BC + 1AC + 1AB & \text{complement} \\
&= BC + AC + AB & \text{identity}
\end{aligned}
$$

- Resulting circuit uses fewer gates.

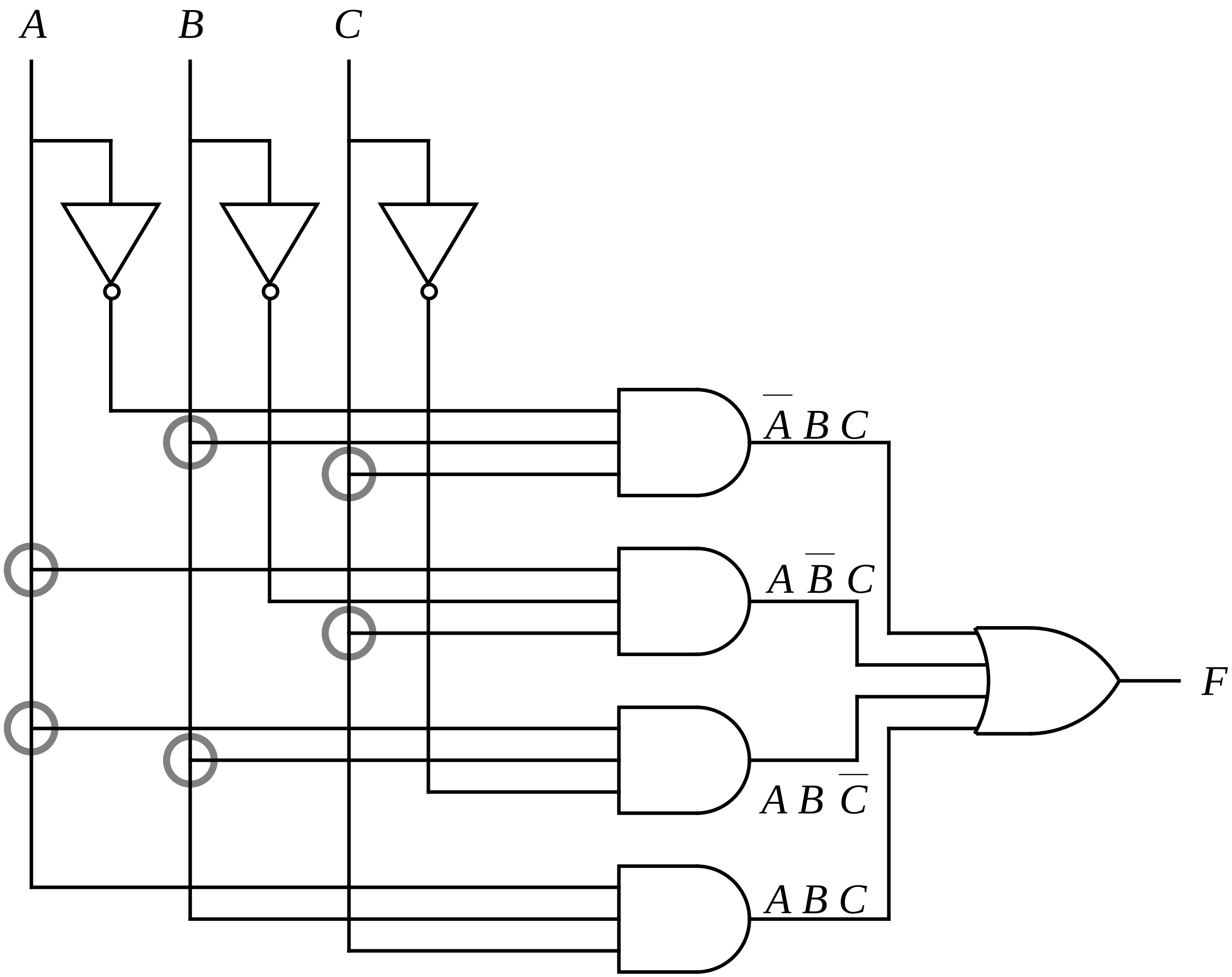
The Algebraic Method

- **This majority circuit is functionally equivalent to the previous majority circuit, but this one is in its minimal two-level form:**



AND-OR Implementation of Majority

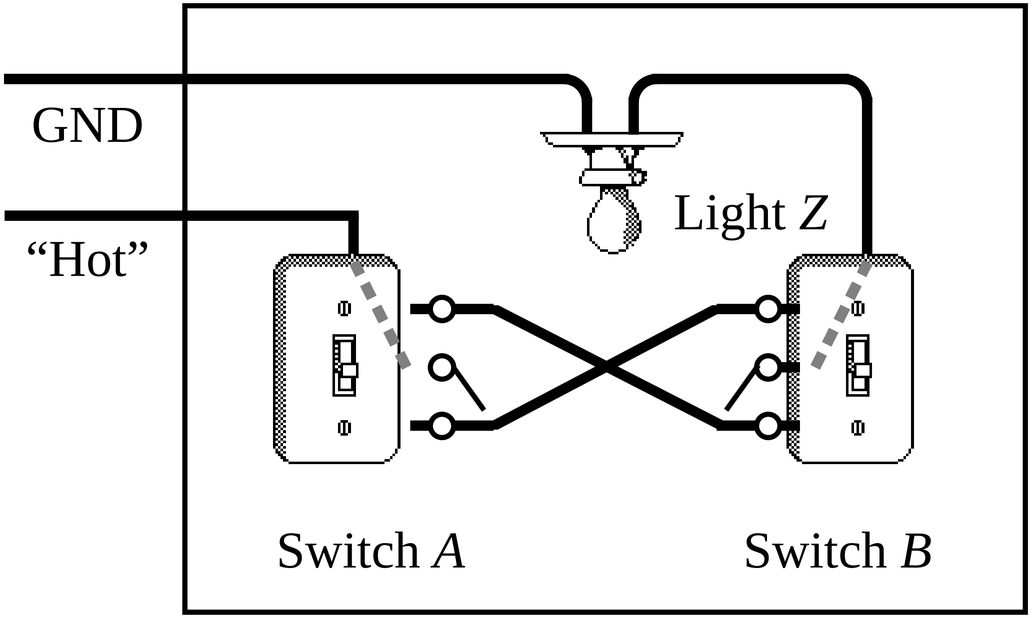
- **Gate count is 8, gate input count is 19.**



How do we make gates???

A Truth Table

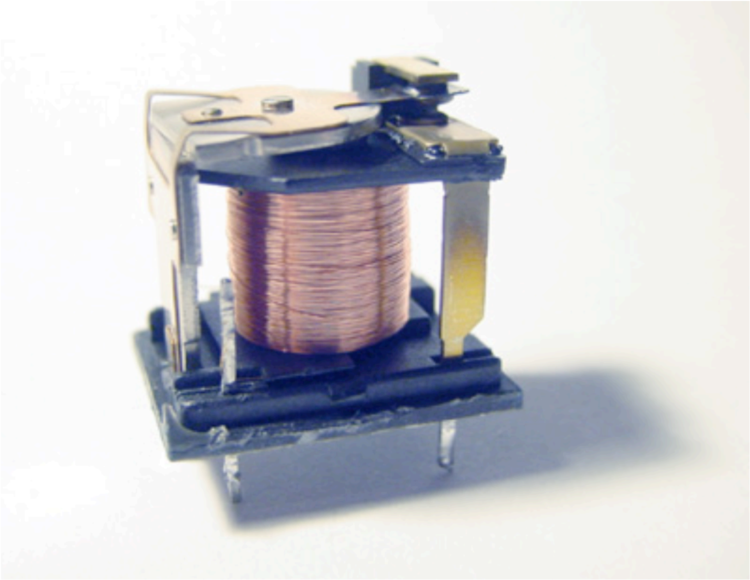
- **Developed in 1854 by George Boole.**
- **Further developed by Claude Shannon (Bell Labs).**
- **Outputs are computed for all possible input combinations (how many input combinations are there?)**
- **Consider a room with two light switches. How must they work?**

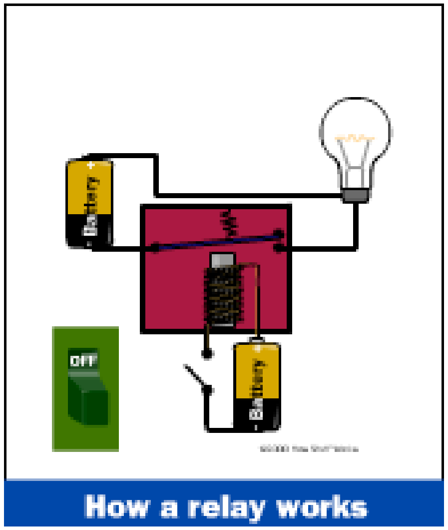


Inputs		Output
A	*B*	*Z*
0	0	0
0	1	1
1	0	1
1	1	0

Electrically Operated Switch

- Example: a relay





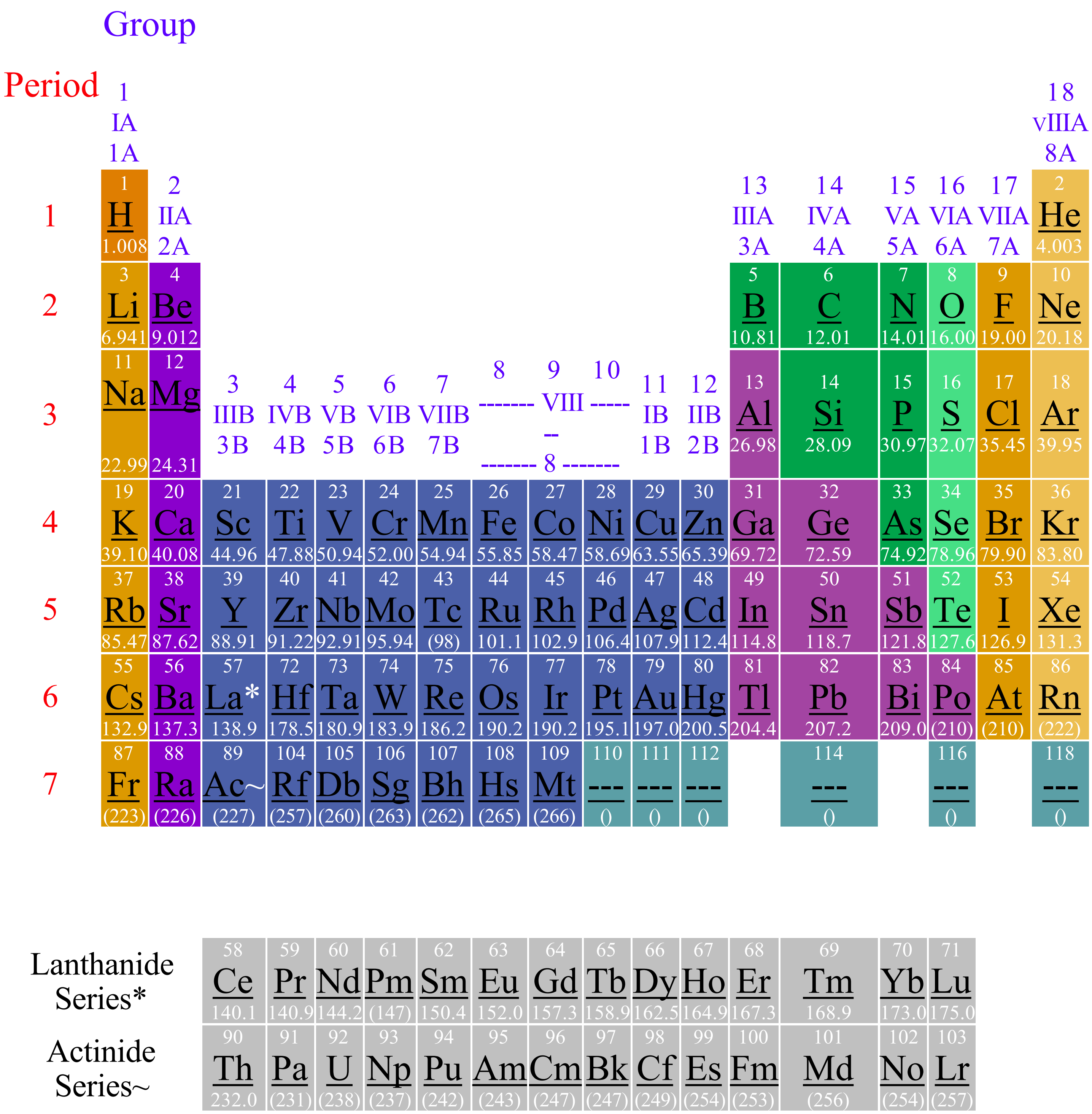
source: http://www.howstuffworks.com/relay.htm

Semiconductors

- Electrical properties of silicon
- Doping: adding impurities to silicon
- Diodes and the P-N junction
- Field-effect transistors

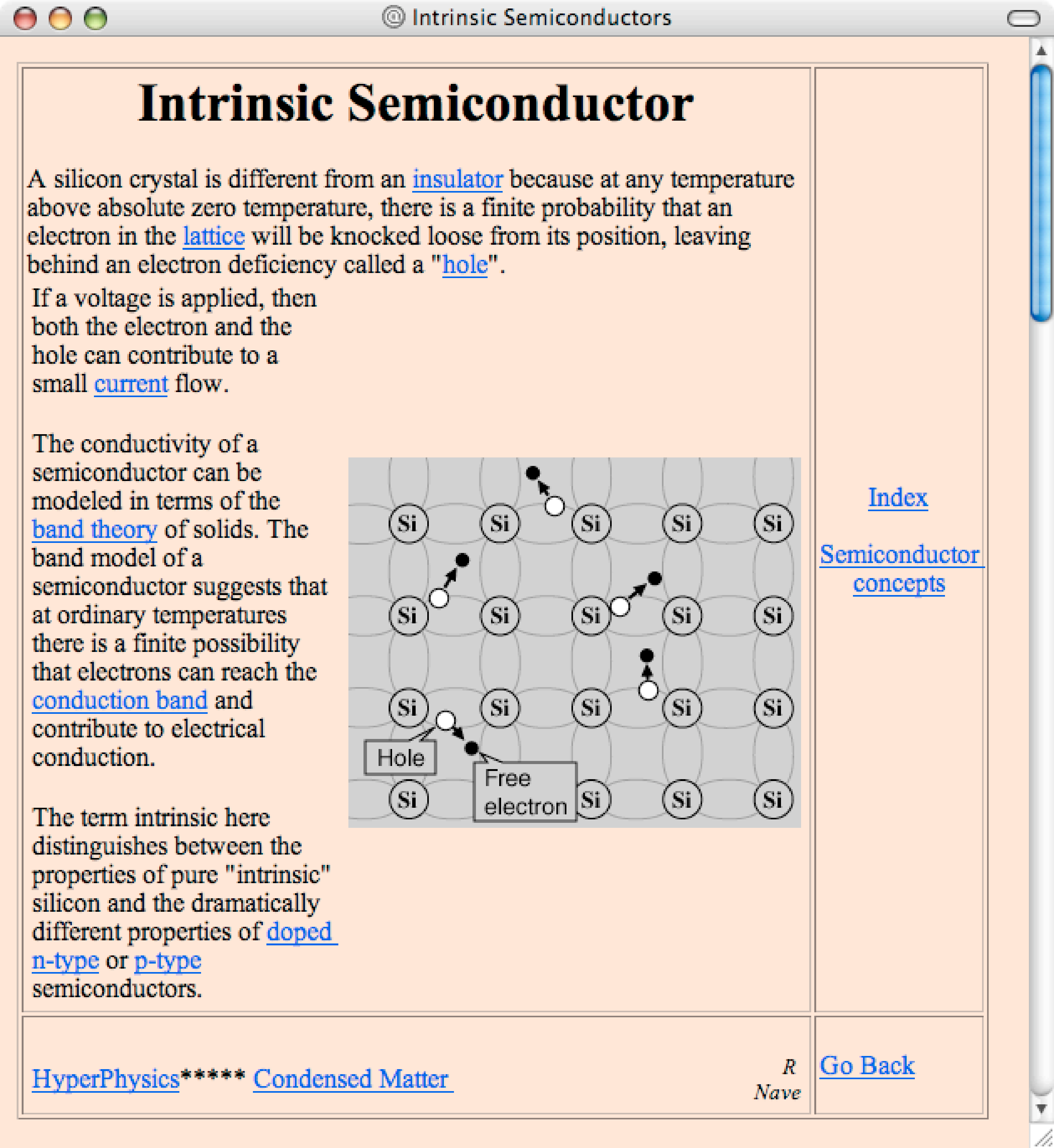
Los Alamos National Laboratory's Chemistry Division Presents

Periodic Table of the Elements



Period \ Group	1 IA 1A	2 IIA 2A	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8	9 VIII 8	10	11 IB 1B	12 IIB 2B	13 IIIA 3A	14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	18 vIIIA 8A
1	1 H 1.008																	2 He 4.003
2	3 Li 6.941	4 Be 9.012											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
3	11 Na 22.99	12 Mg 24.31											13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.07	17 Cl 35.45	18 Ar 39.95
4	19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.88	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.47	28 Ni 58.69	29 Cu 63.55	30 Zn 65.39	31 Ga 69.72	32 Ge 72.59	33 As 74.92	34 Se 78.96	35 Br 79.90	36 Kr 83.80
5	37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92.91	42 Mo 95.94	43 Tc (98)	44 Ru 101.1	45 Rh 102.9	46 Pd 106.4	47 Ag 107.9	48 Cd 112.4	49 In 114.8	50 Sn 118.7	51 Sb 121.8	52 Te 127.6	53 I 126.9	54 Xe 131.3
6	55 Cs 132.9	56 Ba 137.3	57 La* 138.9	72 Hf 178.5	73 Ta 180.9	74 W 183.9	75 Re 186.2	76 Os 190.2	77 Ir 190.2	78 Pt 195.1	79 Au 197.0	80 Hg 200.5	81 Tl 204.4	82 Pb 207.2	83 Bi 209.0	84 Po (210)	85 At (210)	86 Rn (222)
7	87 Fr (223)	88 Ra (226)	89 Ac~ (227)	104 Rf (257)	105 Db (260)	106 Sg (263)	107 Bh (262)	108 Hs (265)	109 Mt (266)	110 --- ()	111 --- ()	112 --- ()		114 --- ()		116 --- ()		118 --- ()

Lanthanide Series*	58 Ce 140.1	59 Pr 140.9	60 Nd 144.2	61 Pm (147)	62 Sm 150.4	63 Eu 152.0	64 Gd 157.3	65 Tb 158.9	66 Dy 162.5	67 Ho 164.9	68 Er 167.3	69 Tm 168.9	70 Yb 173.0	71 Lu 175.0
Actinide Series~	90 Th 232.0	91 Pa (231)	92 U (238)	93 Np (237)	94 Pu (242)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (249)	99 Es (254)	100 Fm (253)	101 Md (256)	102 No (254)	103 Lr (257)



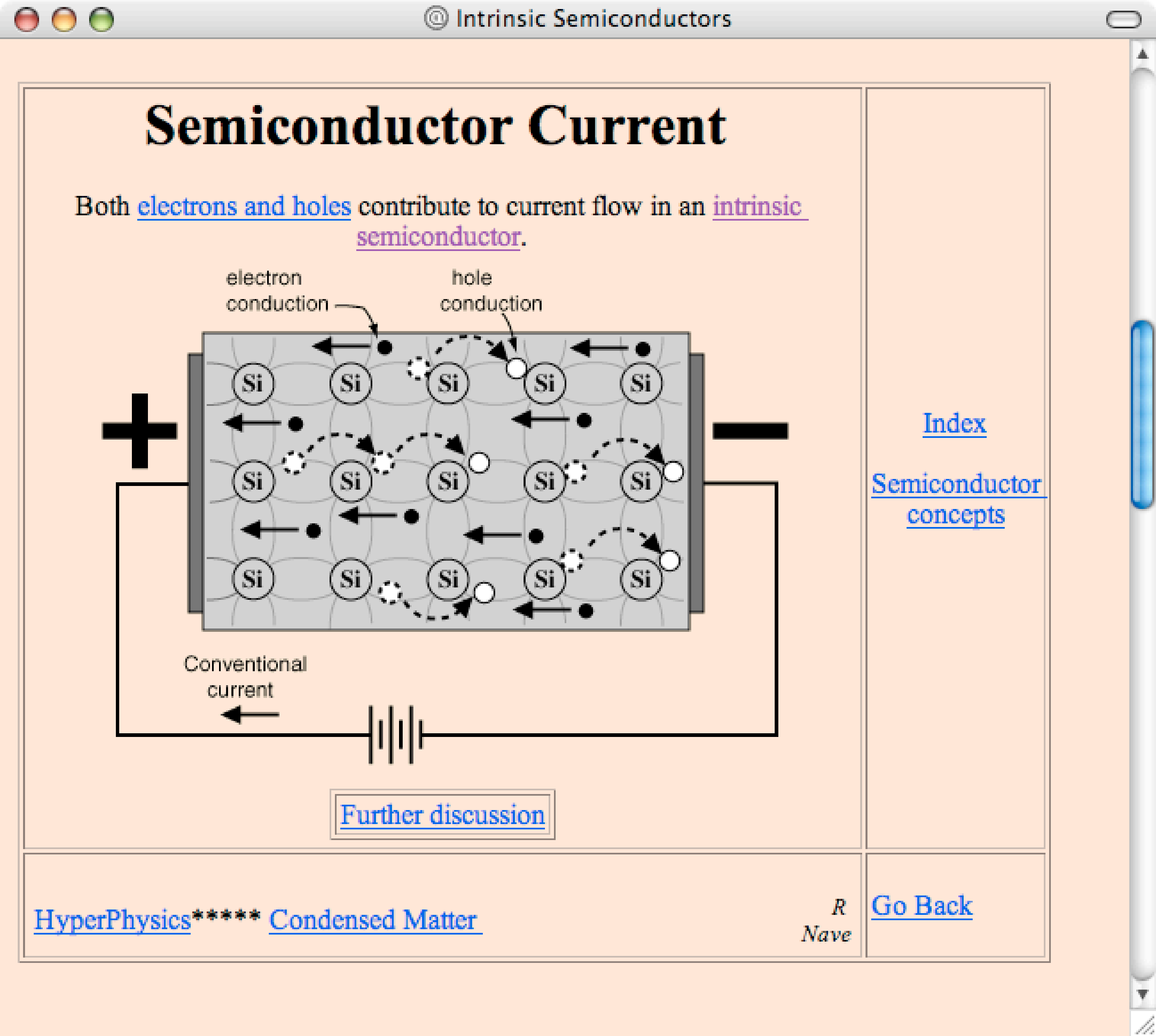
Intrinsic Semiconductor

A silicon crystal is different from an insulator because at any temperature above absolute zero temperature, there is a finite probability that an electron in the lattice will be knocked loose from its position, leaving behind an electron deficiency called a "hole".

If a voltage is applied, then both the electron and the hole can contribute to a small current flow.

The conductivity of a semiconductor can be modeled in terms of the band theory of solids. The band model of a semiconductor suggests that at ordinary temperatures there is a finite possibility that electrons can reach the conduction band and contribute to electrical conduction.

The term intrinsic here distinguishes between the properties of pure "intrinsic" silicon and the dramatically different properties of doped n-type or p-type semiconductors.



Intrinsic Semiconductors

Semiconductor Current

Both electrons and holes contribute to current flow in an intrinsic semiconductor.

Further discussion

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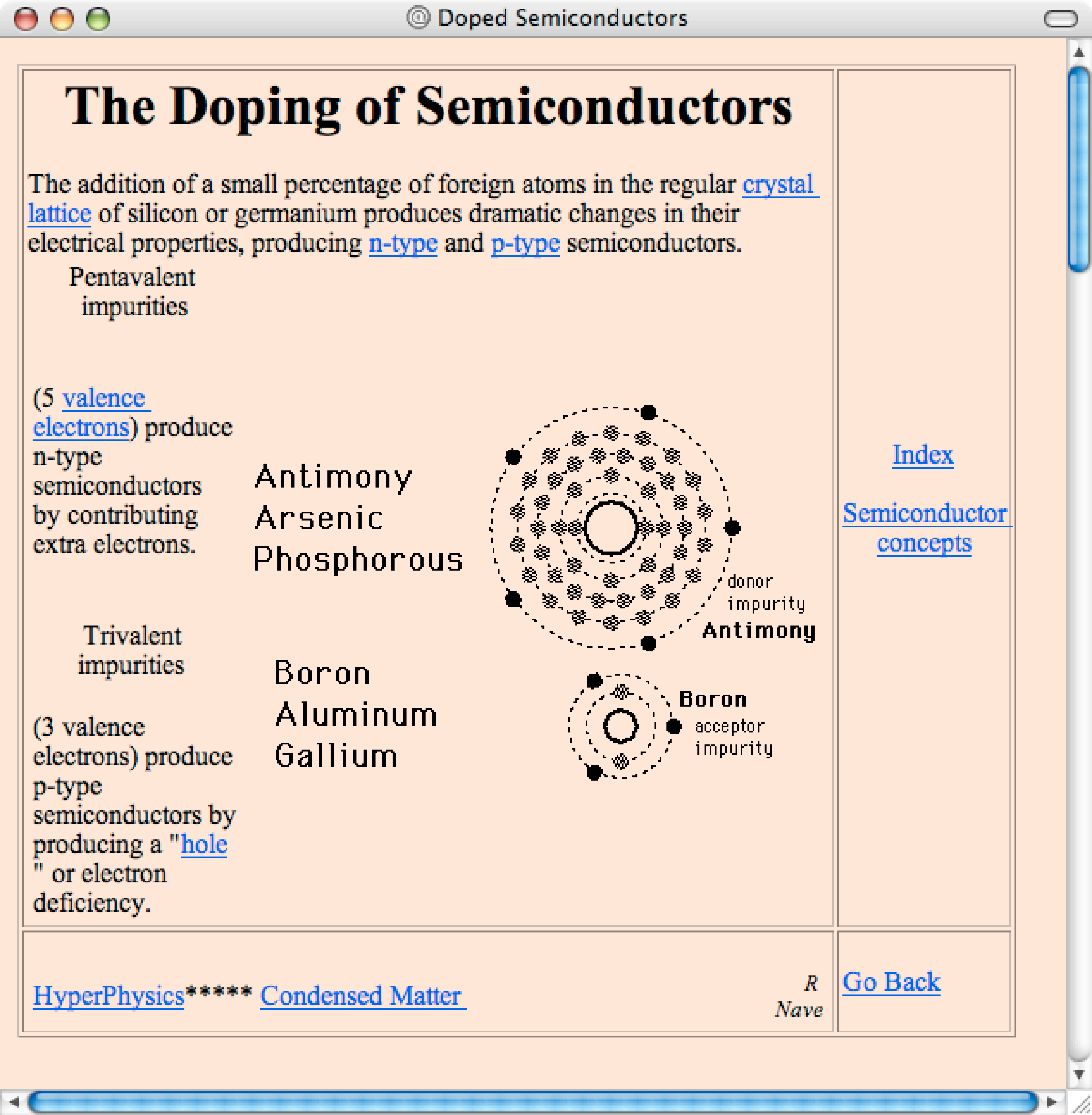

The Doping of Semiconductors

The addition of a small percentage of foreign atoms in the regular crystal lattice of silicon or germanium produces dramatic changes in their electrical properties, producing n-type and p-type semiconductors.

Pentavalent impurities

(5 valence electrons) produce n-type semiconductors by contributing extra electrons.

Antimony
Arsenic
Phosphorous

Trivalent impurities

(3 valence electrons) produce p-type semiconductors by producing a "hole " or electron deficiency.

Boron
Aluminum
Gallium

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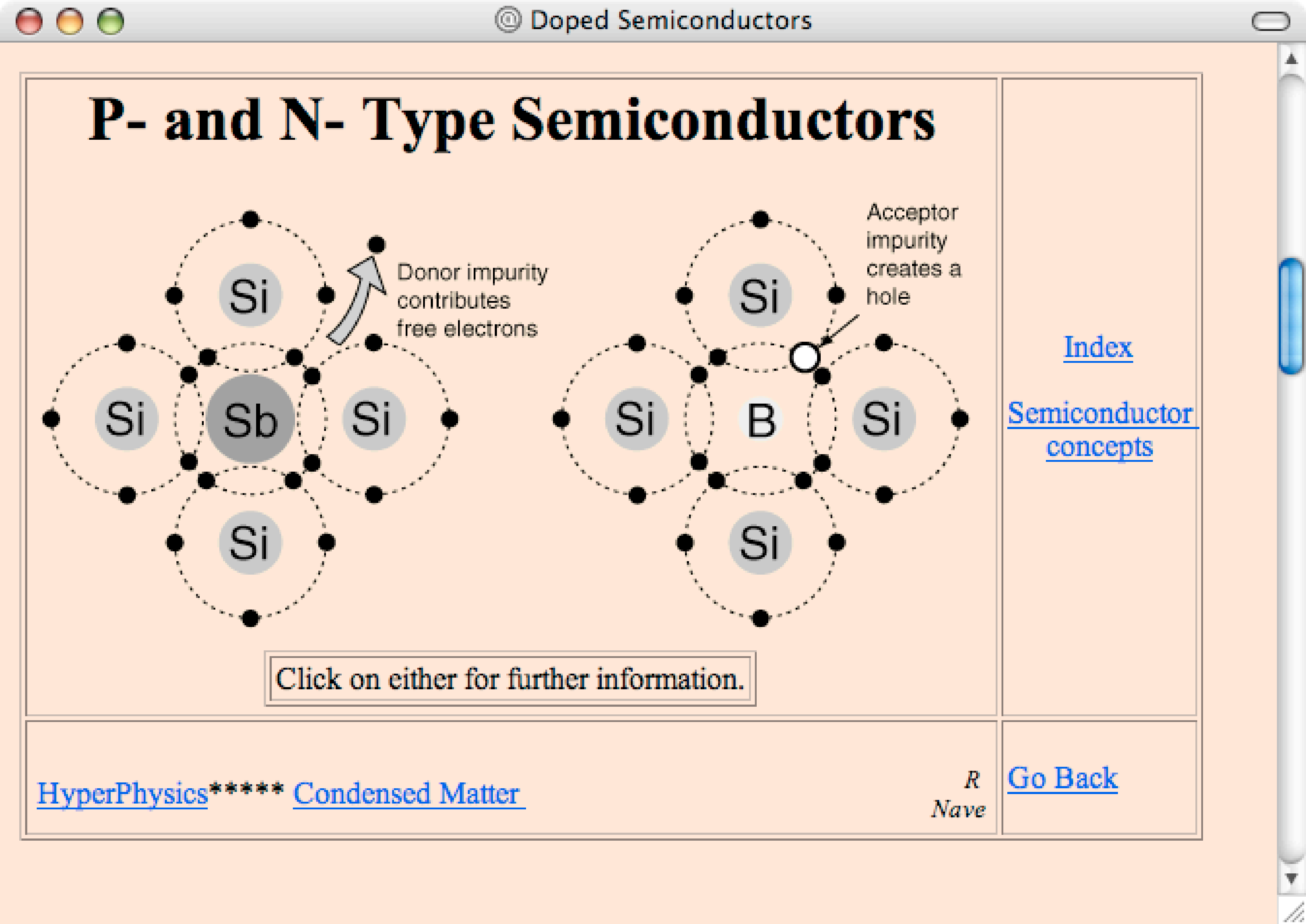

P- and N- Type Semiconductors

Click on either for further information.

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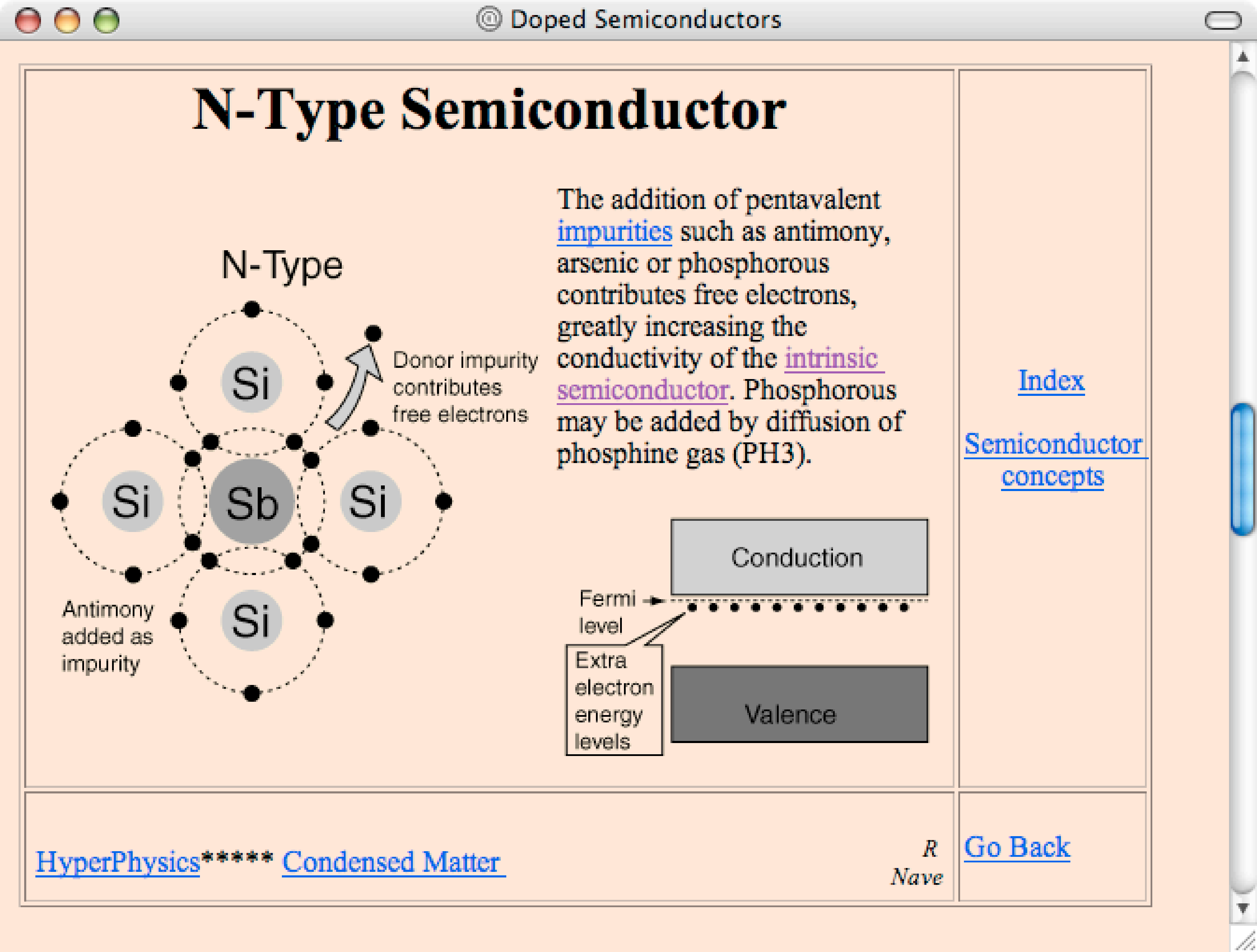
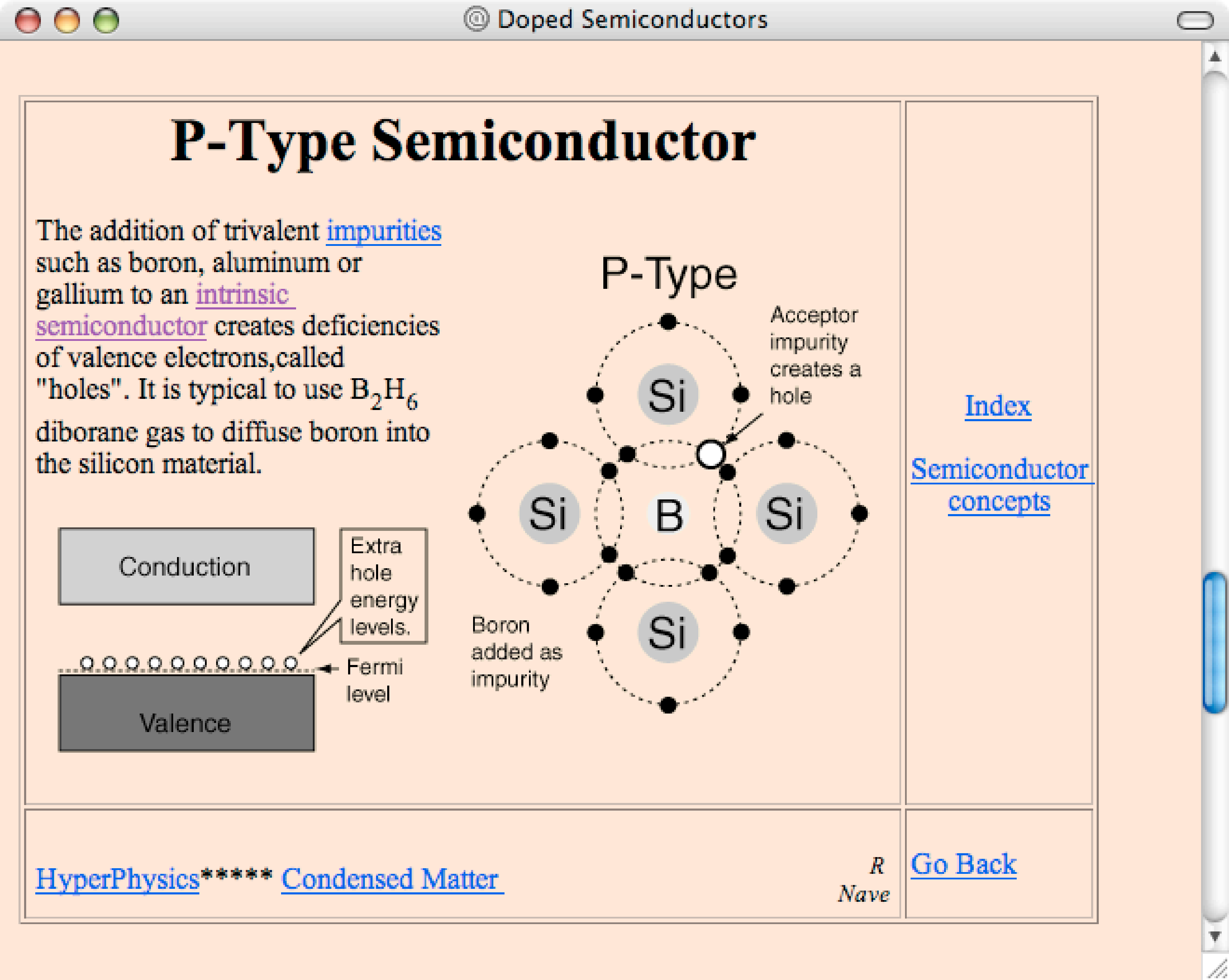


N-Type Semiconductor

The addition of pentavalent impurities such as antimony, arsenic or phosphorous contributes free electrons, greatly increasing the conductivity of the intrinsic semiconductor. Phosphorous may be added by diffusion of phosphine gas (PH3).



P-Type Semiconductor

The addition of trivalent impurities such as boron, aluminum or gallium to an intrinsic semiconductor creates deficiencies of valence electrons,called "holes". It is typical to use B_2H_6 diborane gas to diffuse boron into the silicon material.

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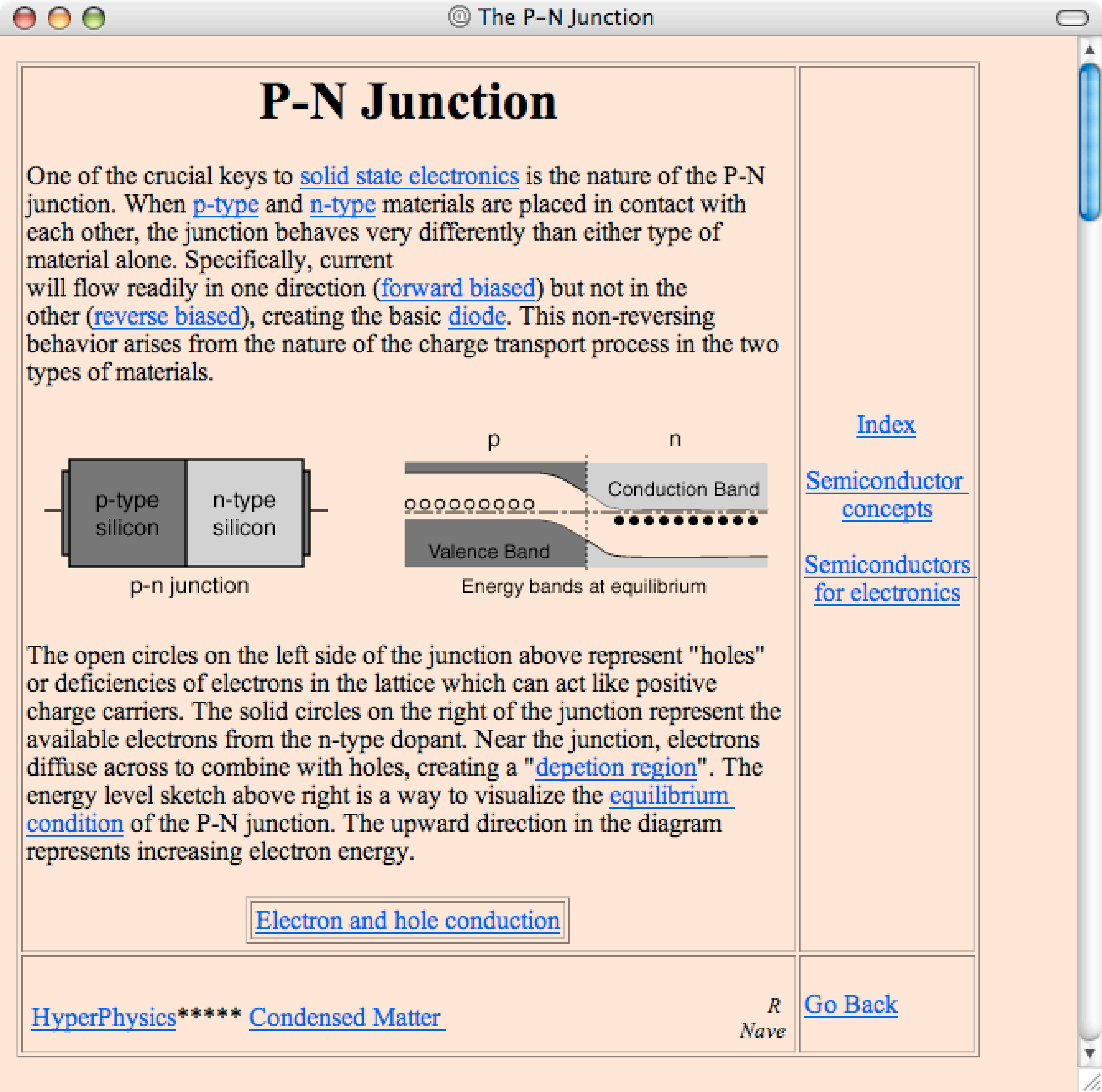

P-N Junction

One of the crucial keys to solid state electronics is the nature of the P-N junction. When p-type and n-type materials are placed in contact with each other, the junction behaves very differently than either type of material alone. Specifically, current will flow readily in one direction (forward biased) but not in the other (reverse biased), creating the basic diode. This non-reversing behavior arises from the nature of the charge transport process in the two types of materials.

p-n junction

Energy bands at equilibrium

The open circles on the left side of the junction above represent "holes" or deficiencies of electrons in the lattice which can act like positive charge carriers. The solid circles on the right of the junction represent the available electrons from the n-type dopant. Near the junction, electrons diffuse across to combine with holes, creating a "depetion region". The energy level sketch above right is a way to visualize the equilibrium condition of the P-N junction. The upward direction in the diagram represents increasing electron energy.

Electron and hole conduction

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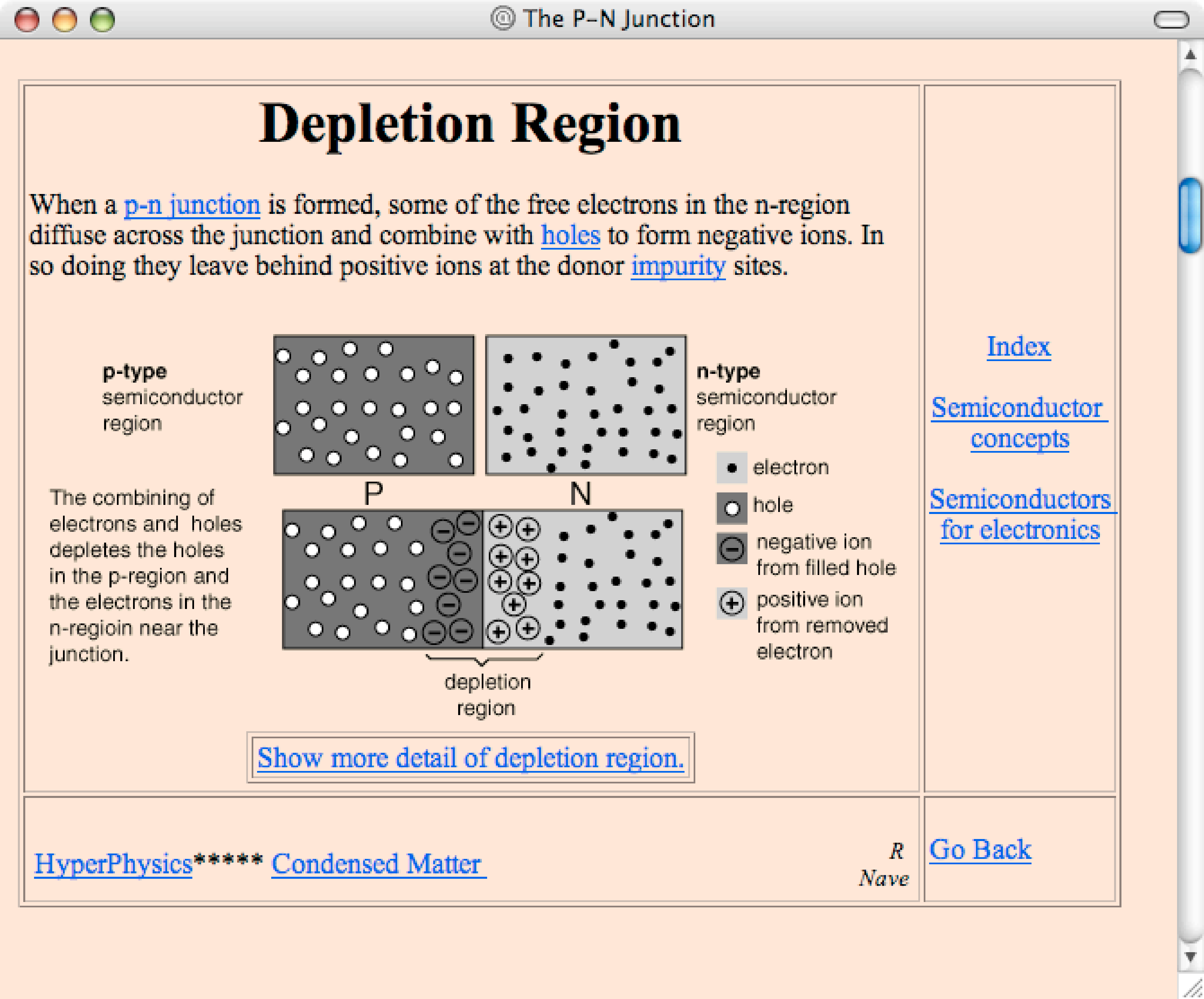

Depletion Region

When a p-n junction is formed, some of the free electrons in the n-region diffuse across the junction and combine with holes to form negative ions. In so doing they leave behind positive ions at the donor impurity sites.

The combining of electrons and holes depletes the holes in the p-region and the electrons in the n-regioin near the junction.

Show more detail of depletion region.

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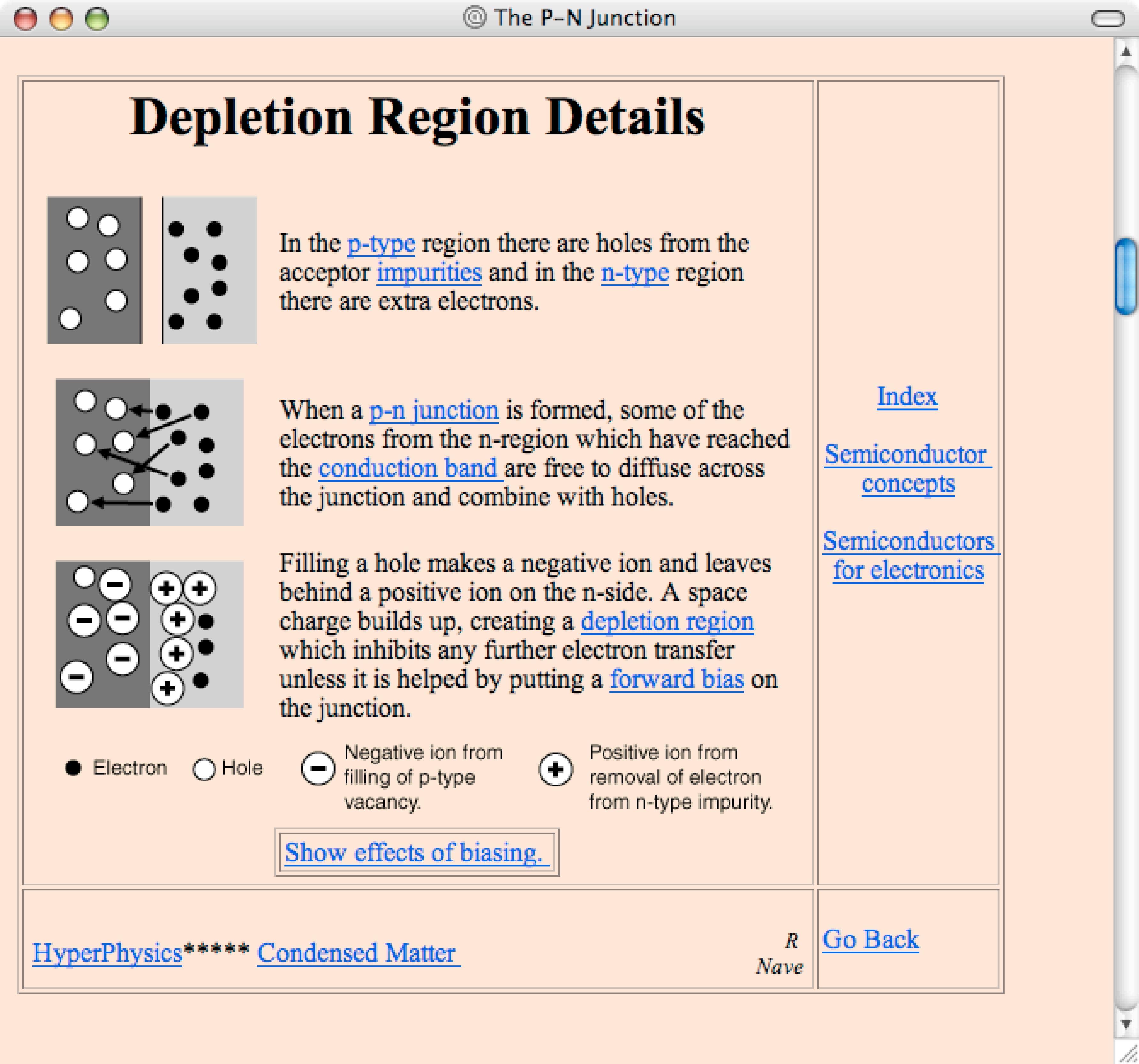


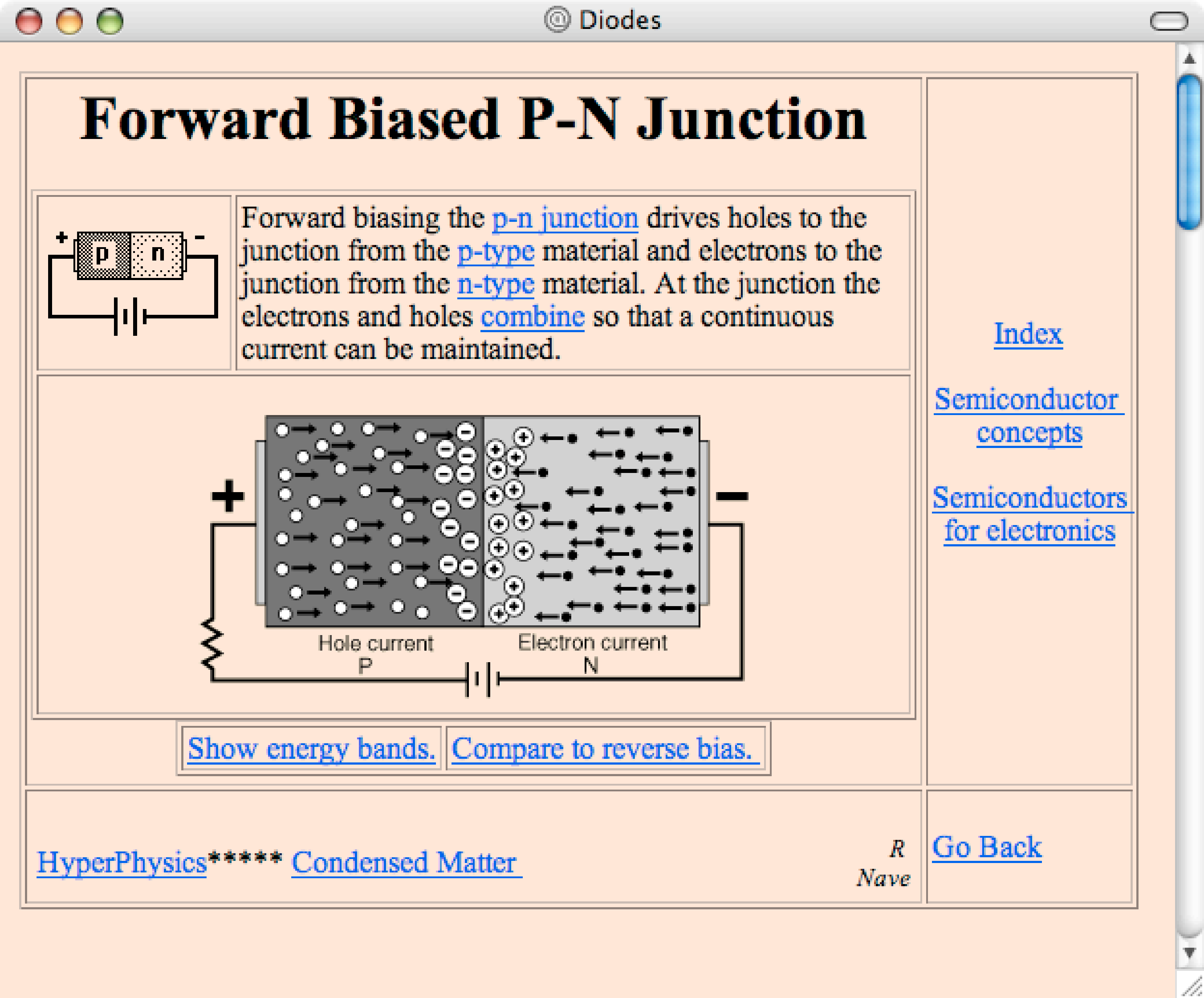
Depletion Region Details

In the p-type region there are holes from the acceptor impurities and in the n-type region there are extra electrons.

When a p-n junction is formed, some of the electrons from the n-region which have reached the conduction band are free to diffuse across the junction and combine with holes.

Filling a hole makes a negative ion and leaves behind a positive ion on the n-side. A space charge builds up, creating a depletion region which inhibits any further electron transfer unless it is helped by putting a forward bias on the junction.

● Electron ○ Hole ⊖ Negative ion from filling of p-type vacancy. ⊕ Positive ion from removal of electron from n-type impurity.



Forward Biased P-N Junction

Forward biasing the p-n junction drives holes to the junction from the p-type material and electrons to the junction from the n-type material. At the junction the electrons and holes combine so that a continuous current can be maintained.

Show energy bands. Compare to reverse bias.

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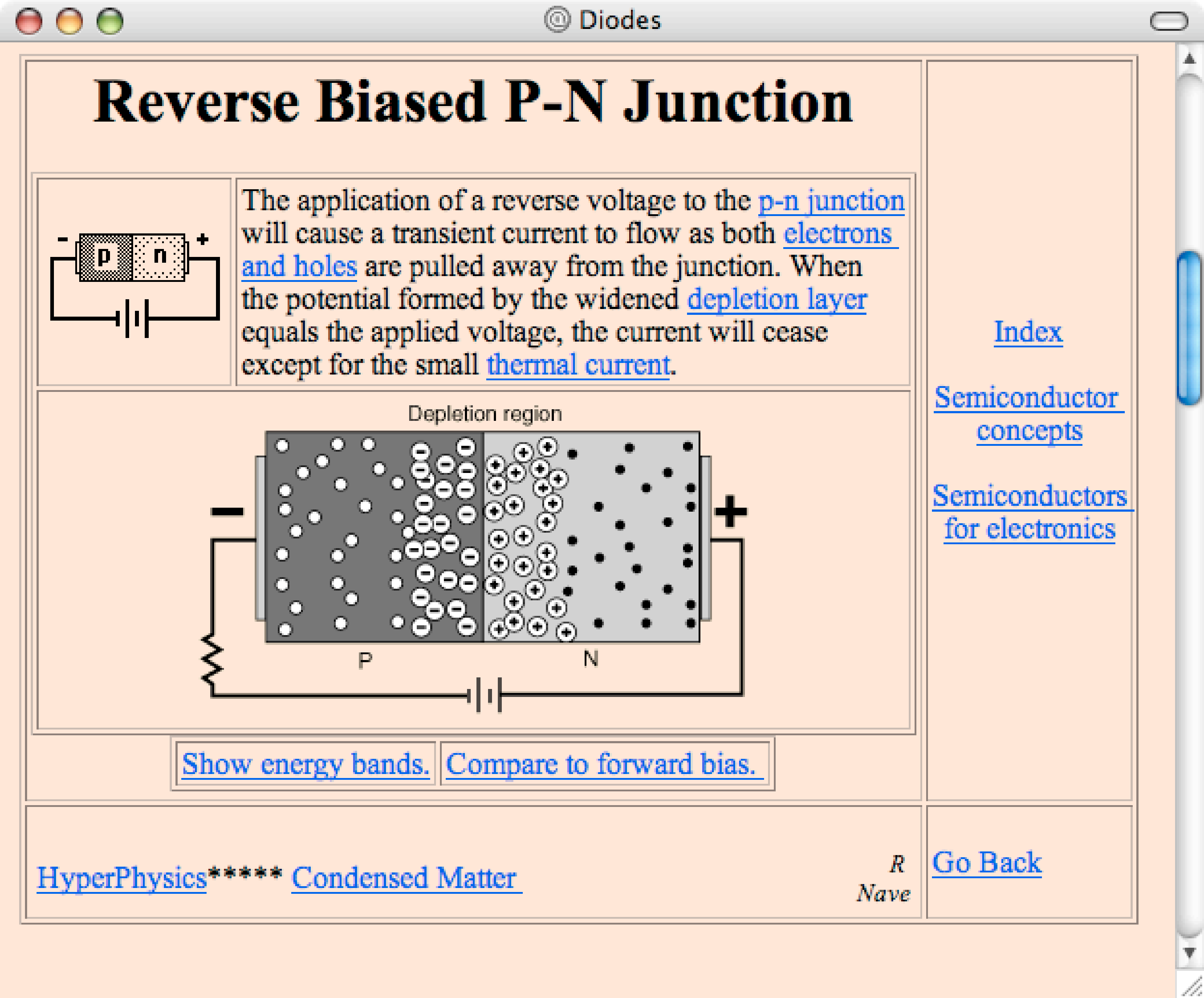

Reverse Biased P-N Junction

The application of a reverse voltage to the p-n junction will cause a transient current to flow as both electrons and holes are pulled away from the junction. When the potential formed by the widened depletion layer equals the applied voltage, the current will cease except for the small thermal current.

Show energy bands. Compare to forward bias.

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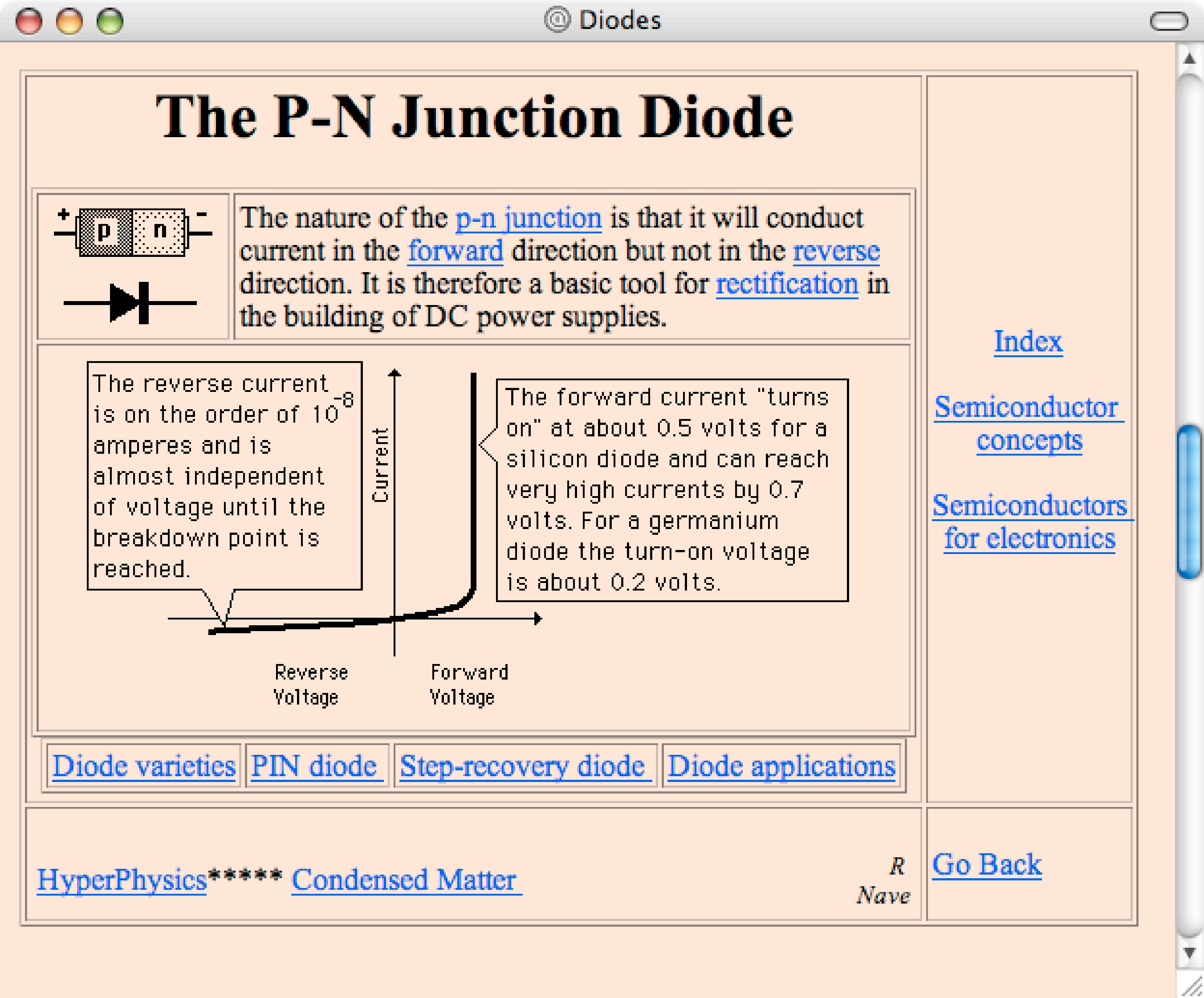

The P-N Junction Diode

The nature of the p-n junction is that it will conduct current in the forward direction but not in the reverse direction. It is therefore a basic tool for rectification in the building of DC power supplies.

The reverse current is on the order of 10^{-8} amperes and is almost independent of voltage until the breakdown point is reached.

The forward current "turns on" at about 0.5 volts for a silicon diode and can reach very high currents by 0.7 volts. For a germanium diode the turn-on voltage is about 0.2 volts.

Diode varieties | PIN diode | Step-recovery diode | Diode applications

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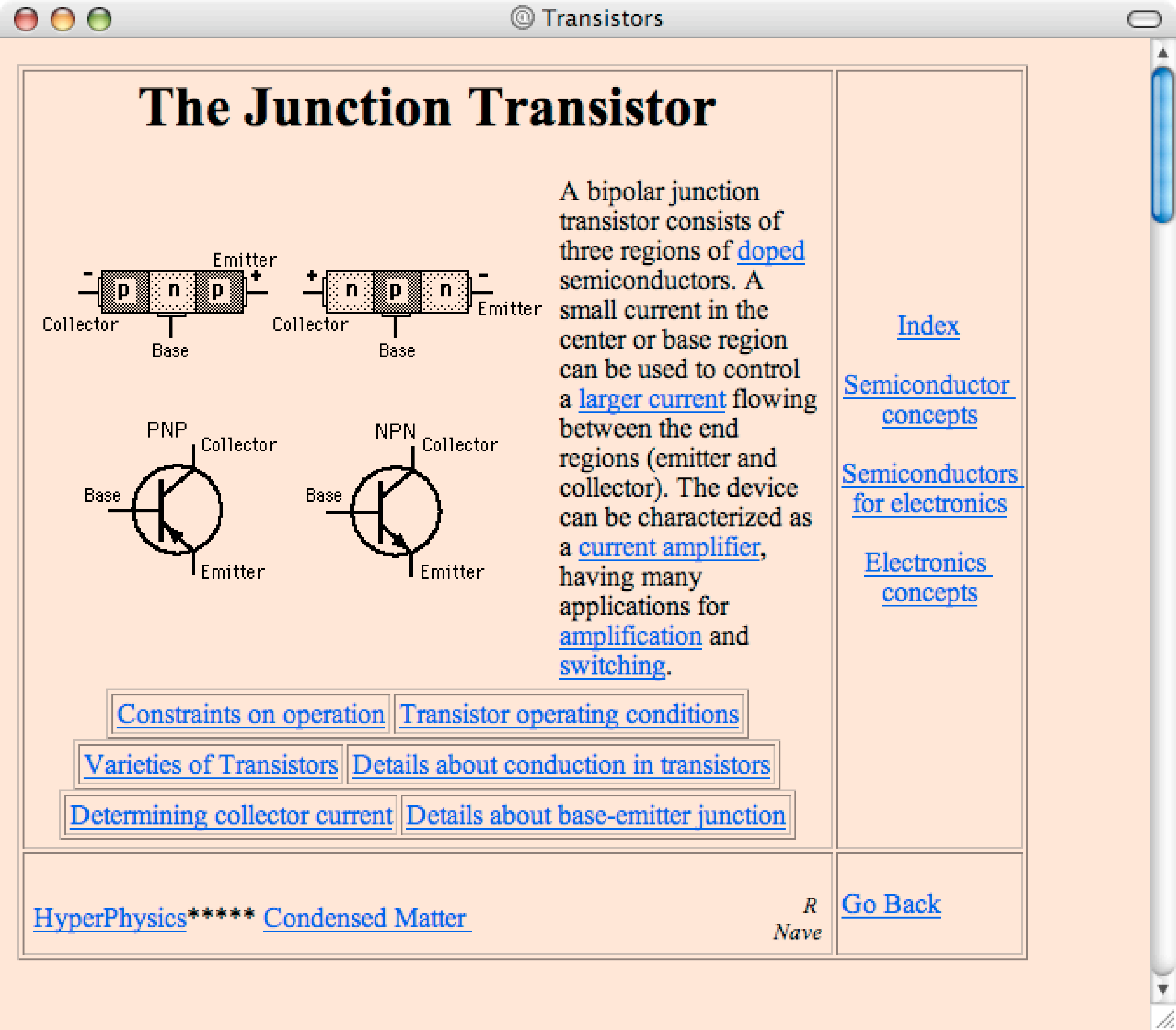

The Junction Transistor

A bipolar junction transistor consists of three regions of doped semiconductors. A small current in the center or base region can be used to control a larger current flowing between the end regions (emitter and collector). The device can be characterized as a current amplifier, having many applications for amplification and switching.

Constraints on operation	Transistor operating conditions
Varieties of Transistors	Details about conduction in transistors
Determining collector current	Details about base-emitter junction

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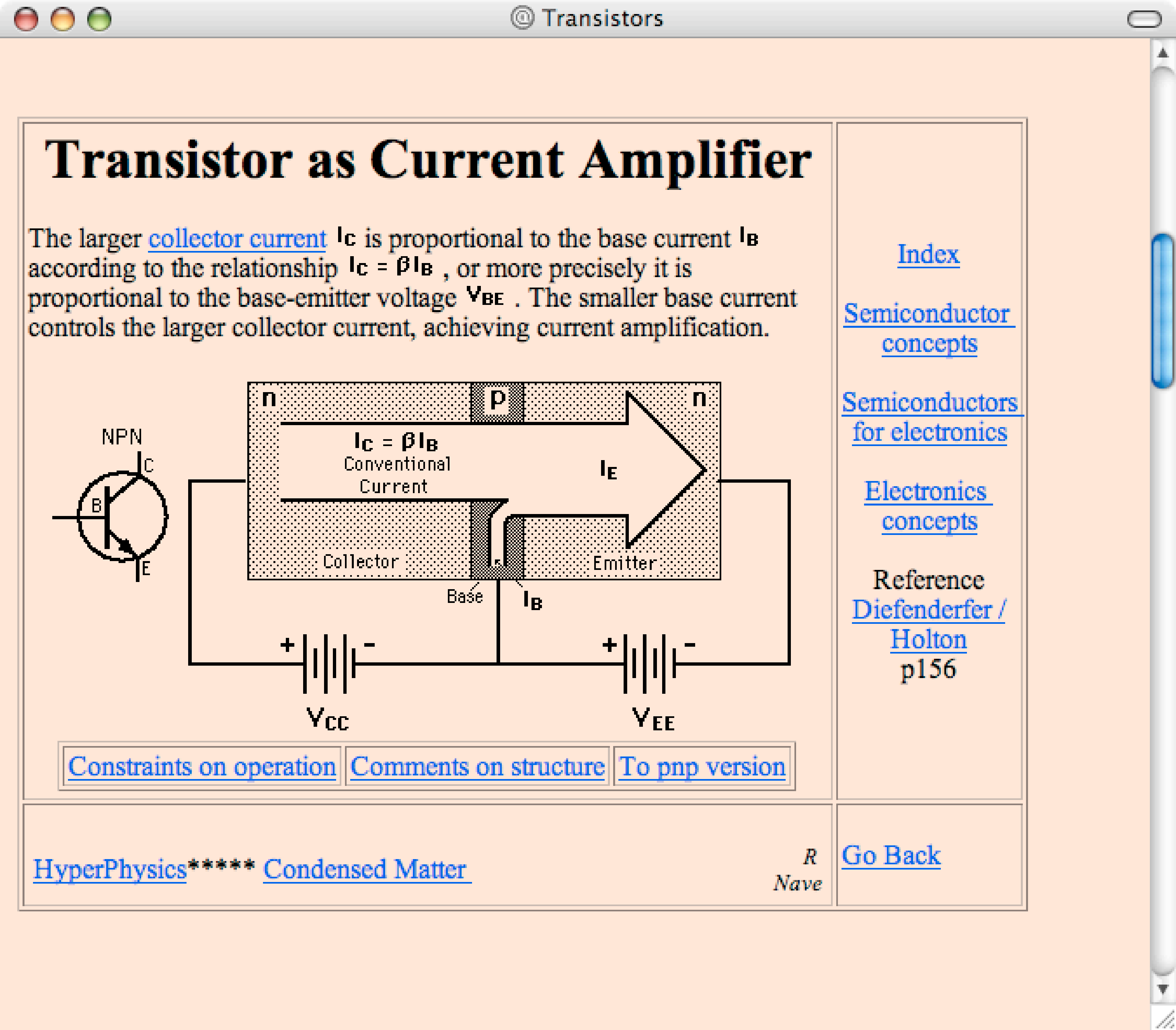

Transistor as Current Amplifier

The larger collector current I_C is proportional to the base current I_B according to the relationship $I_C = \beta I_B$, or more precisely it is proportional to the base-emitter voltage V_{BE} . The smaller base current controls the larger collector current, achieving current amplification.

Constraints on operation | Comments on structure | To pnp version

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Reference
Diefenderfer / Holton
p156

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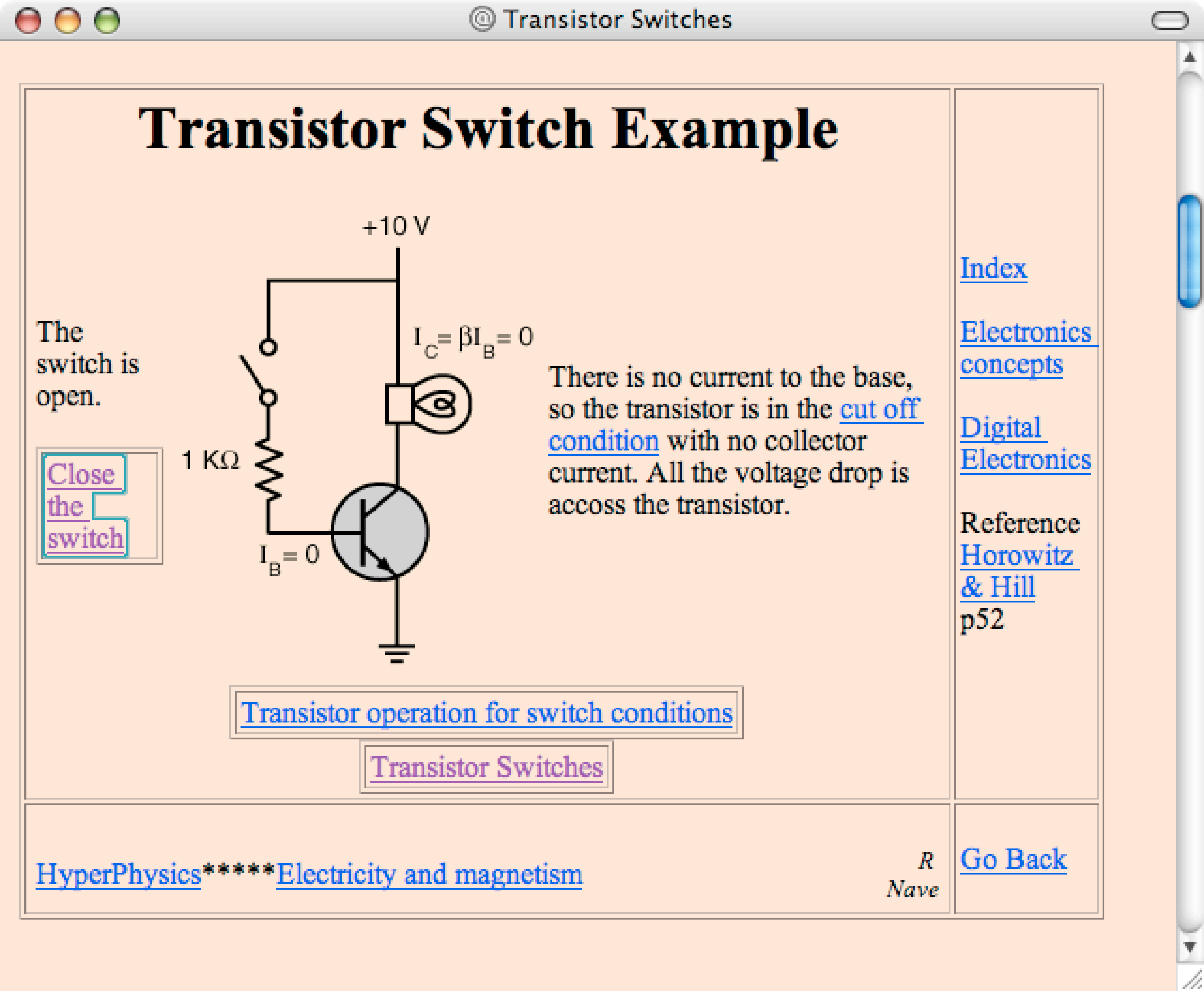

Transistor Switch Example

The switch is open.

Close the switch

There is no current to the base, so the transistor is in the cut off condition with no collector current. All the voltage drop is accoss the transistor.

Transistor operation for switch conditions

Transistor Switches

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Digital Electronics

Reference Horowitz & Hill p52

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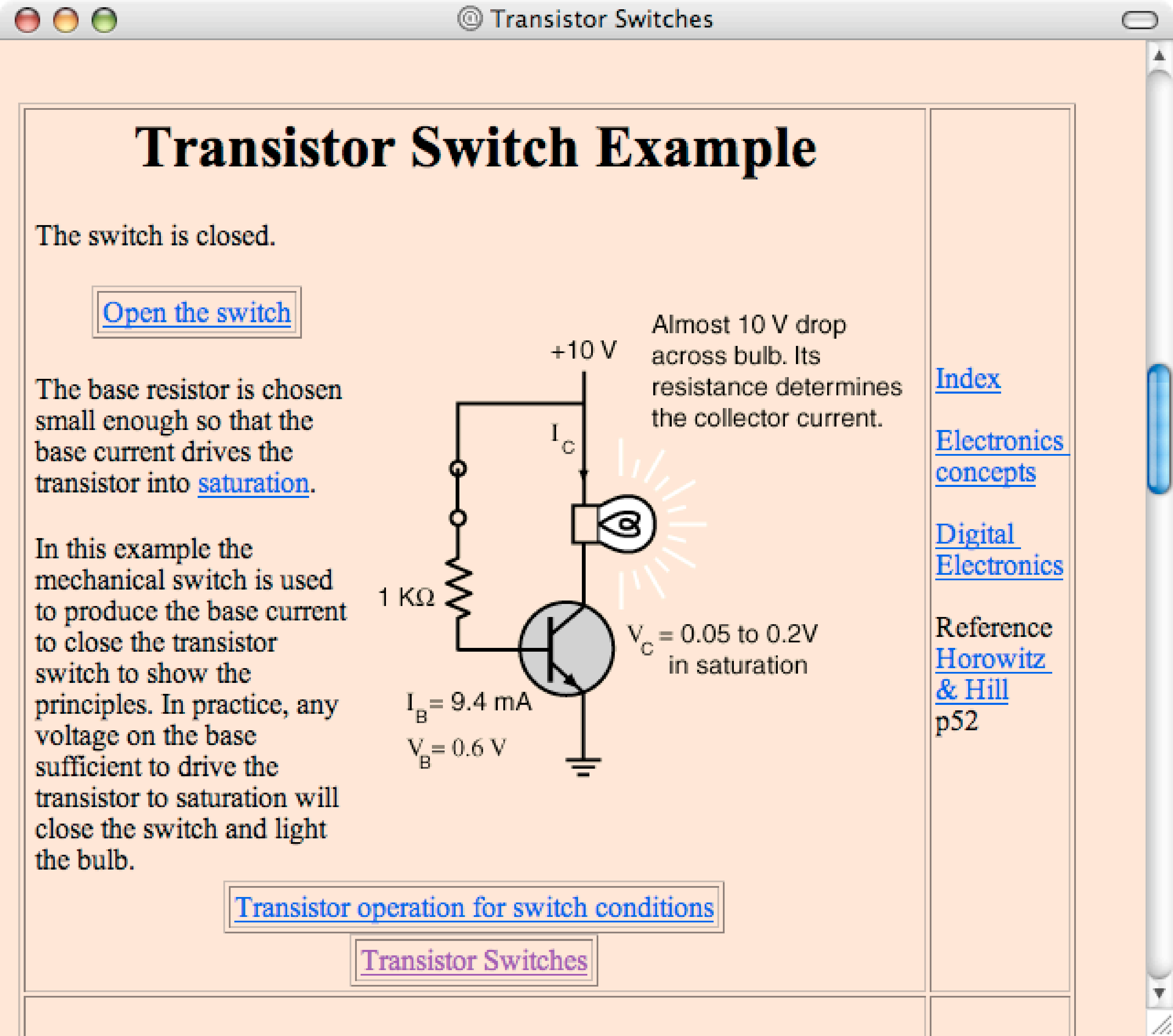

Transistor Switch Example

The switch is closed.

Open the switch

The base resistor is chosen small enough so that the base current drives the transistor into saturation.

In this example the mechanical switch is used to produce the base current to close the transistor switch to show the principles. In practice, any voltage on the base sufficient to drive the transistor to saturation will close the switch and light the bulb.

Almost 10 V drop across bulb. Its resistance determines the collector current.

Transistor operation for switch conditions

Transistor Switches

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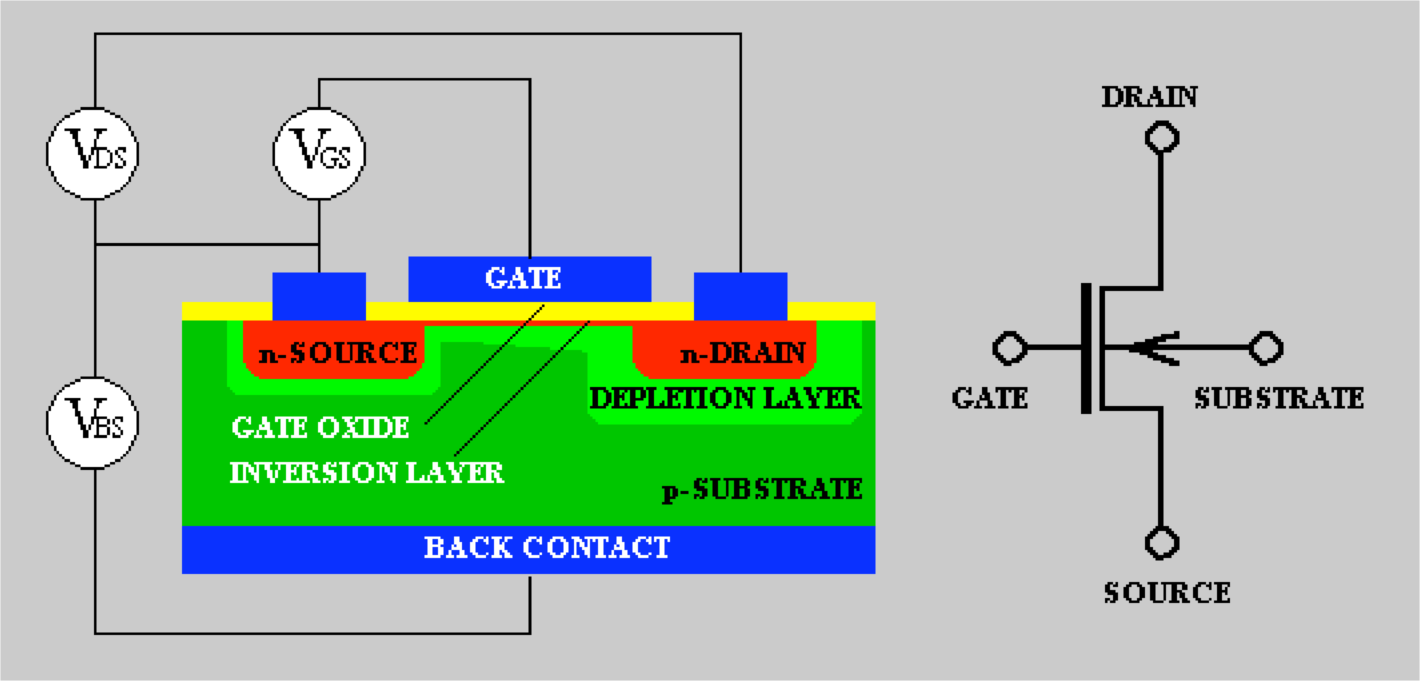
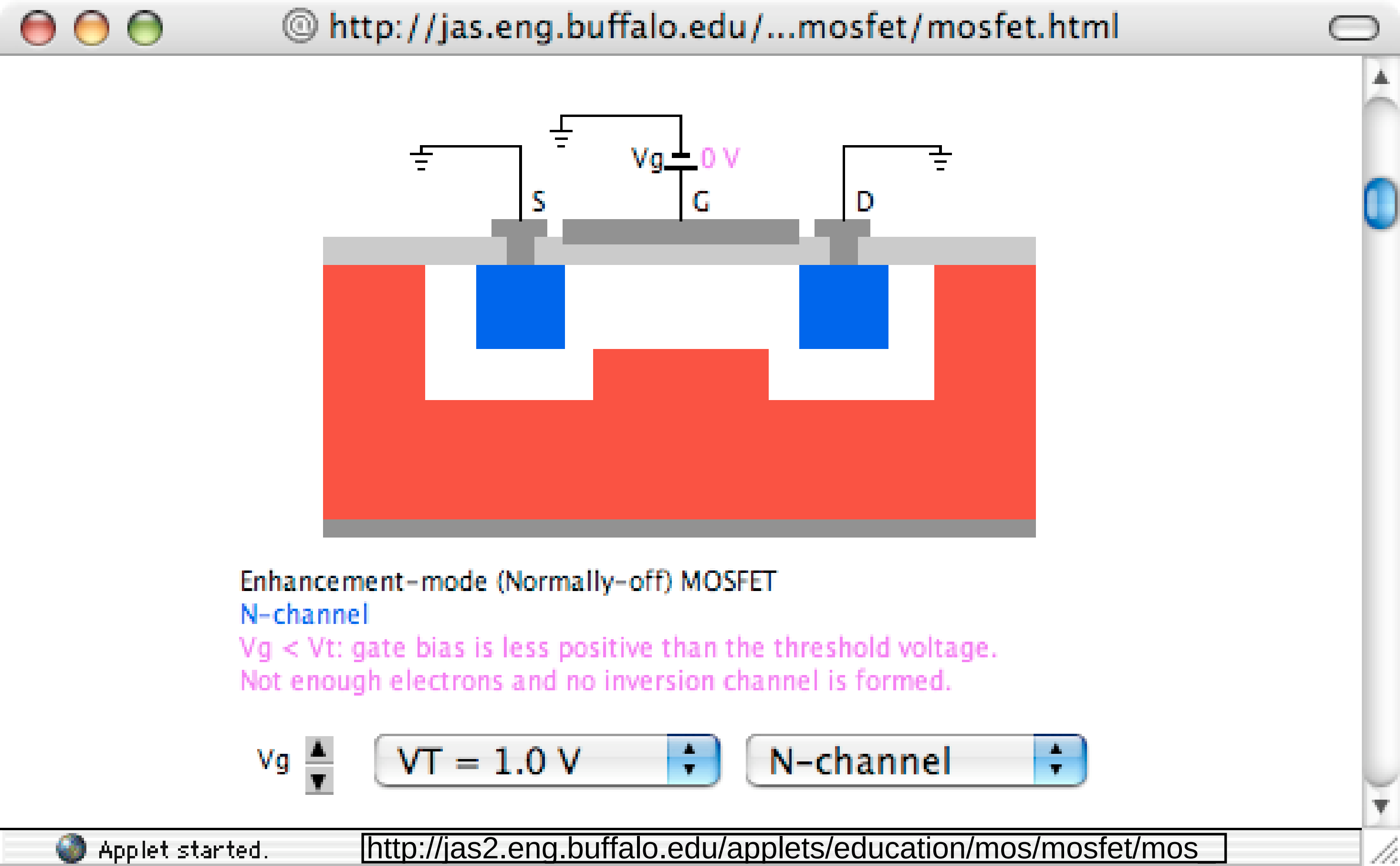
Reference Horowitz & Hill p52



from: http://ece-www.colorado.edu/~bart/book/mosintro.htm

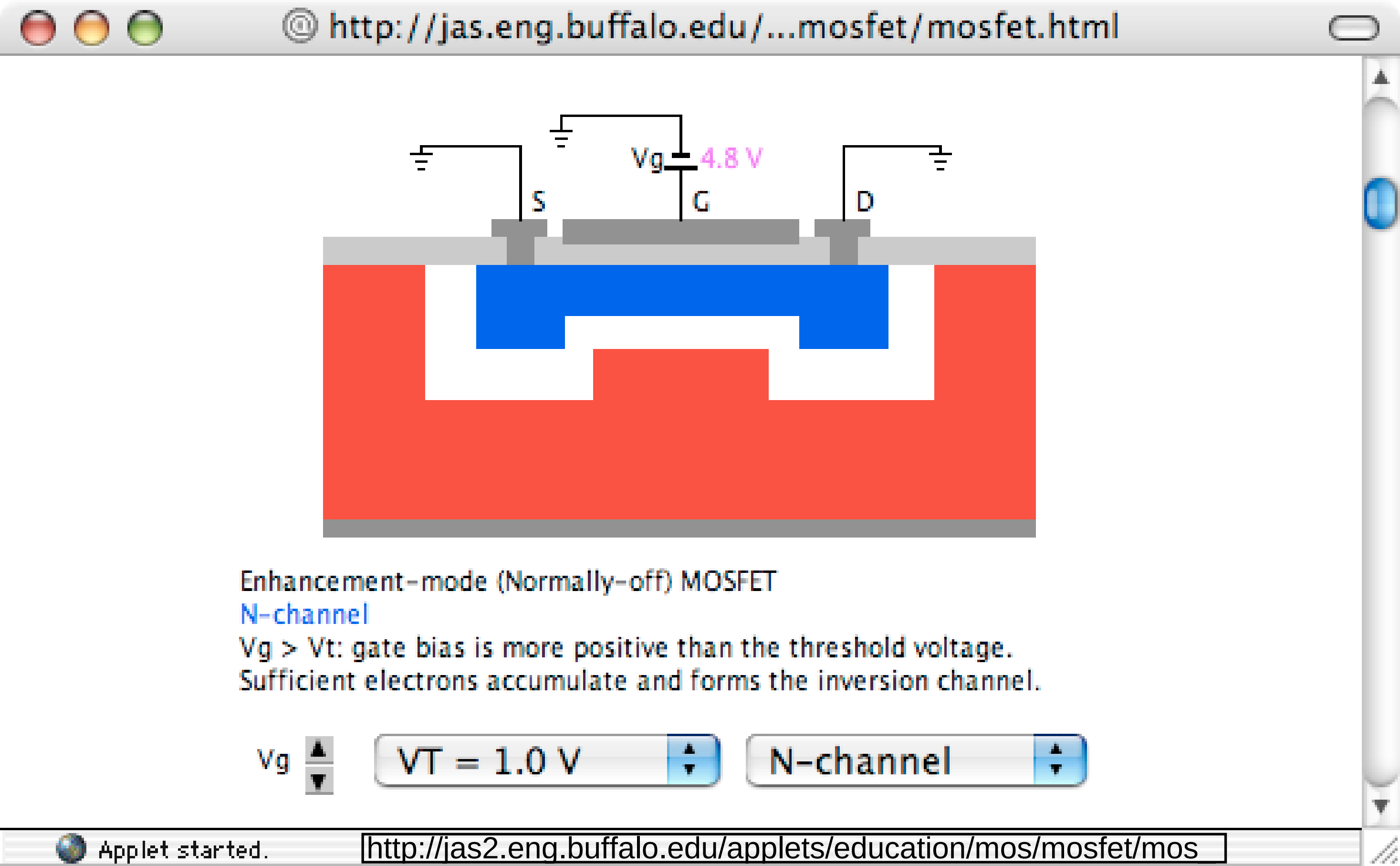


http://jas.eng.buffalo.edu/...mosfet/mosfet.html

Enhancement-mode (Normally-off) MOSFET
N-channel
Vg < Vt: gate bias is less positive than the threshold voltage.
Not enough electrons and no inversion channel is formed.

VT = 1.0 V
N-channel

Applet started.
http://jas2.eng.buffalo.edu/applets/education/mos/mosfet/mos



http://jas.eng.buffalo.edu/...mosfet/mosfet.html

Enhancement-mode (Normally-off) MOSFET
N-channel
Vg > Vt: gate bias is more positive than the threshold voltage.
Sufficient electrons accumulate and forms the inversion channel.

VT = 1.0 V

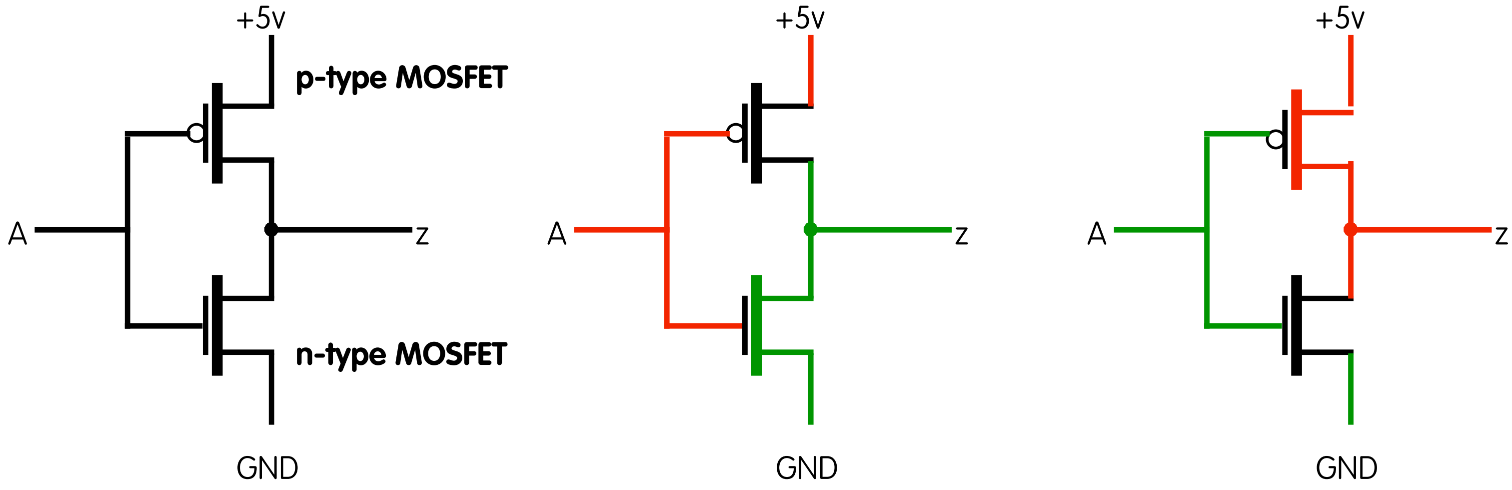
N-channel

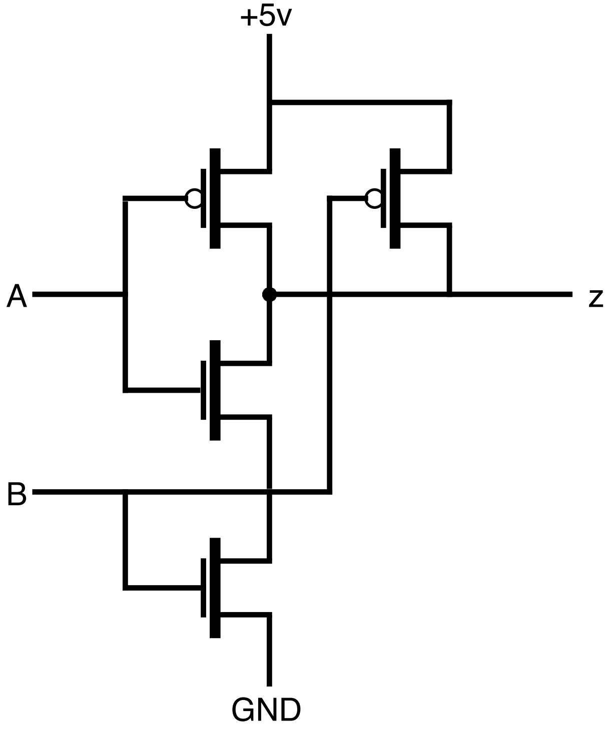
Applet started.

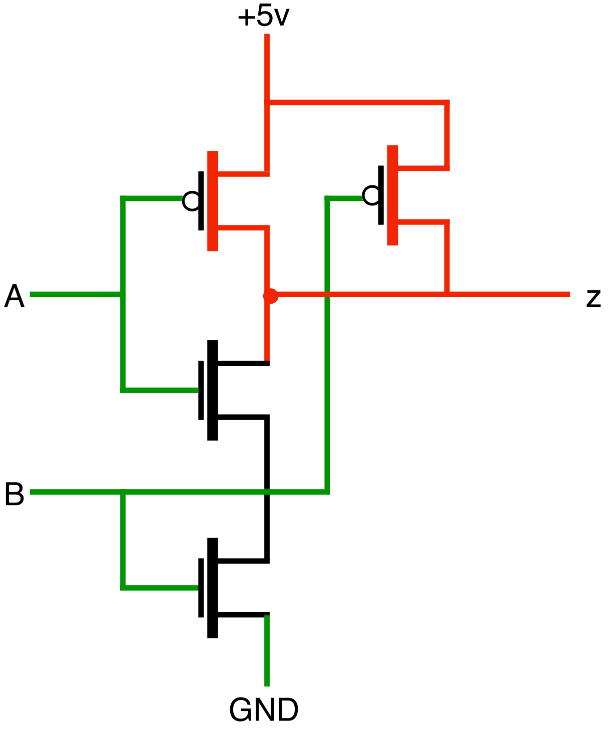
http://jas2.eng.buffalo.edu/applets/education/mos/mosfet/mos

An Inverter using MOSFET

- CMOS = complementary metal oxide semiconductor
- P-type transistor conducts when gate is low
- N-type transistor conducts when gate is high

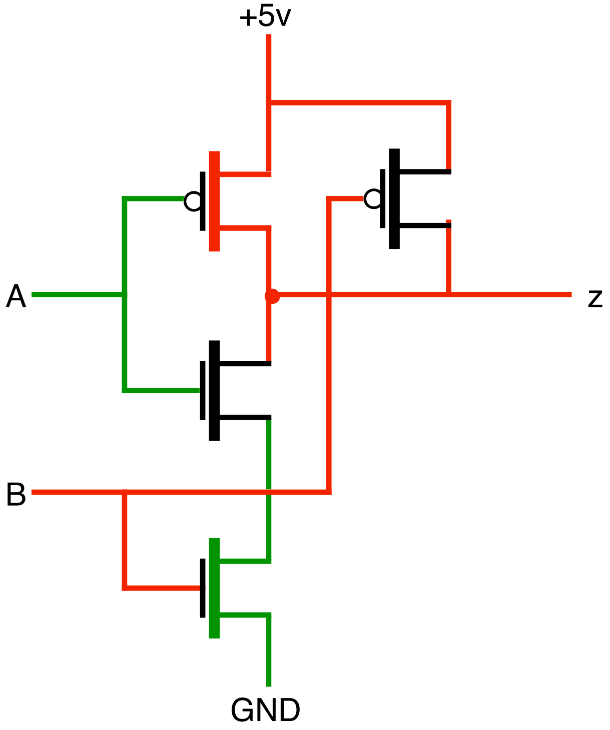


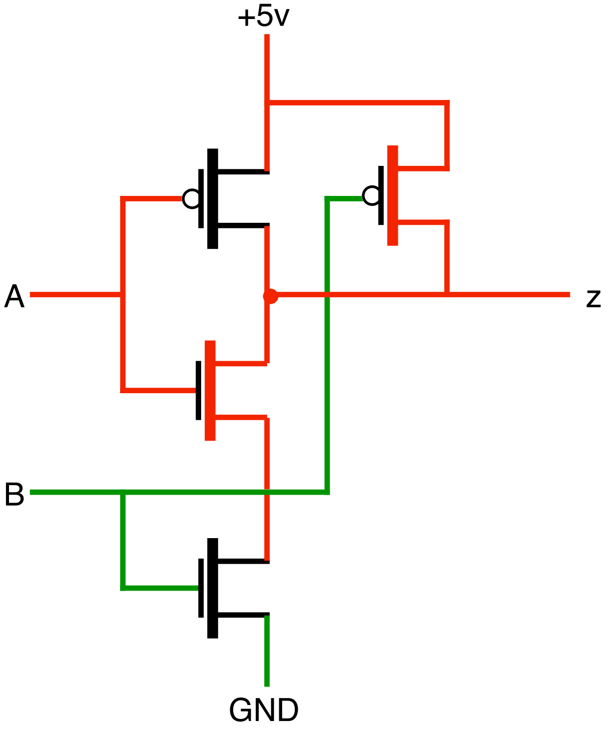


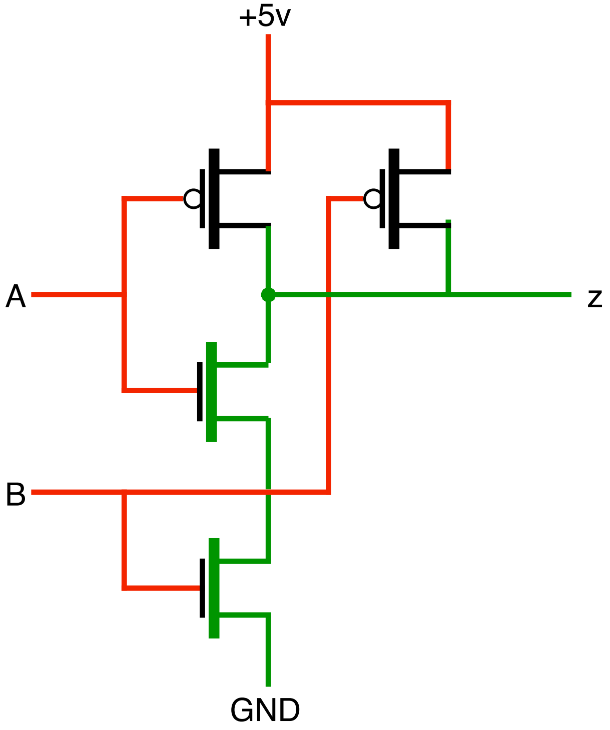


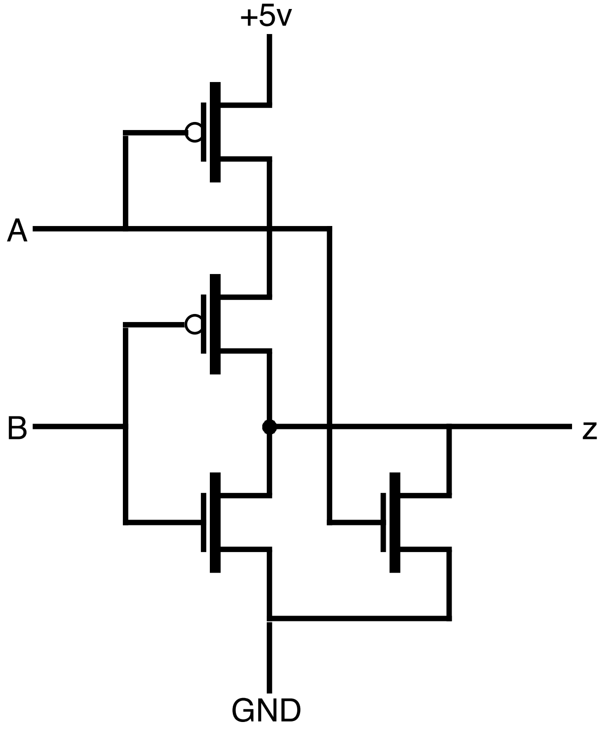
NAND GATE

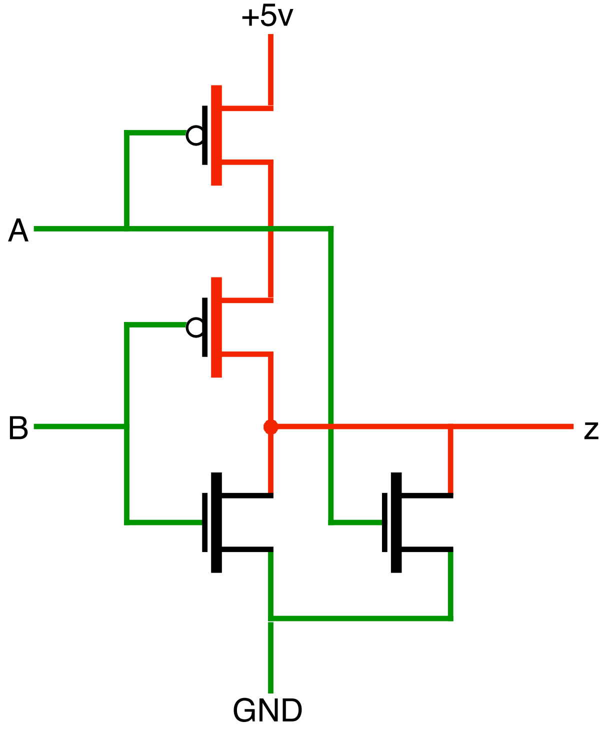
A	B	z
0	0	1
0	1	1
1	0	1
1	1	0





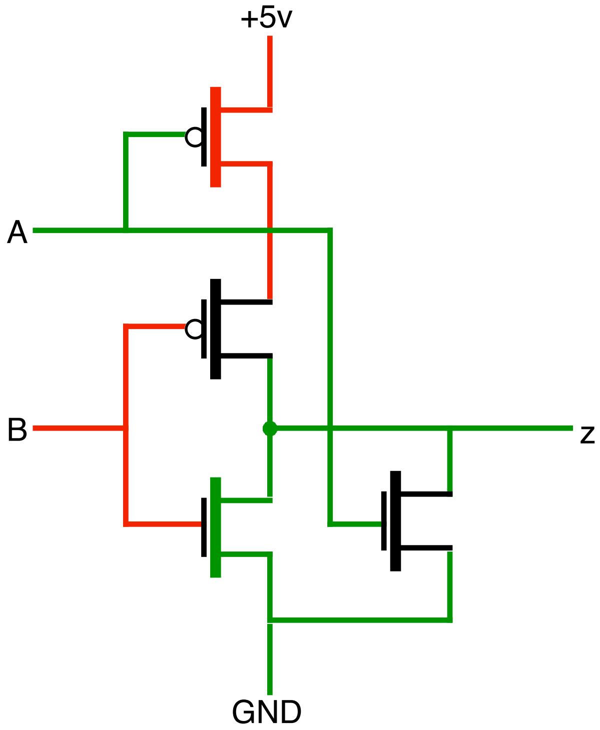


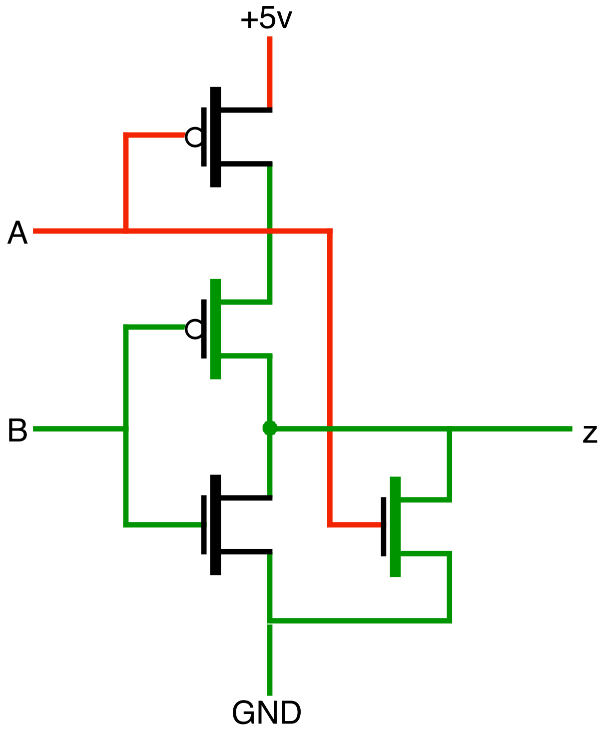


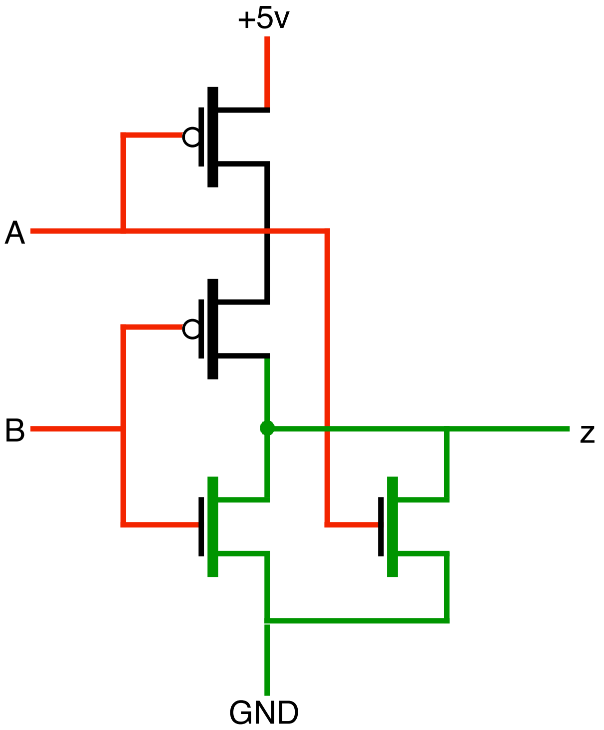


NOR GATE

A	B	z
0	0	1
0	1	0
1	0	0
1	1	0







CMOS Logic vs Bipolar Logic

- MOSFET transistors are easier to miniaturize
- CMOS logic has lower current drain
- CMOS logic is easier to manufacture

Next time

- Circuits for Addition

References

- Materials on semiconductors, PN junction and transistors taken from the HyperPhysics web site:

  <http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html>